2.2 Các Mô Hình Kênh Vô Tuyến Di Động Ngoài Trời

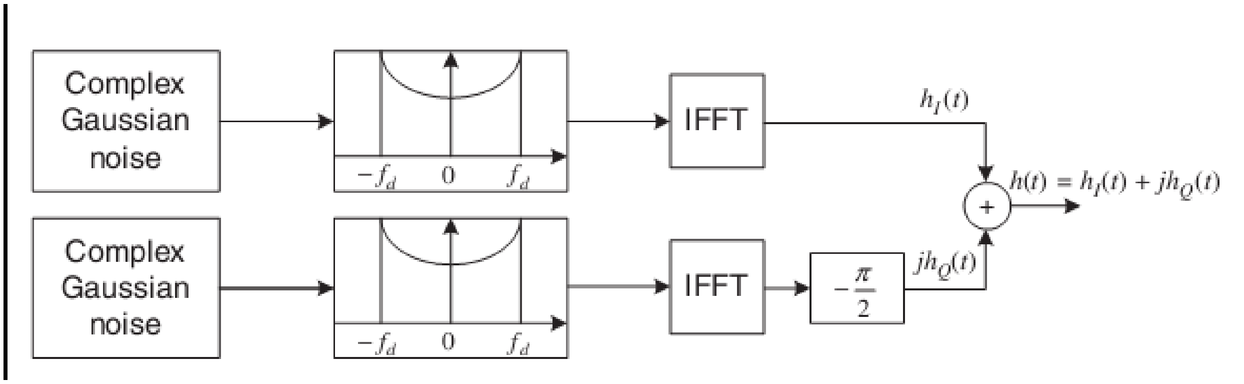
Khác với bản chất tĩnh hoặc bán tĩnh của kênh trong nhà, các kênh ngoài trời thường có đặc trưng là độ lợi kênh biến đổi theo thời gian, tùy thuộc vào tốc độ di động của thiết bị đầu cuối. Tùy thuộc vào tốc độ di động, sự thay đổi theo thời gian của độ lợi kênh bị chi phối bởi phổ Doppler, phổ này xác định tương quan miền thời gian trong độ lợi kênh Trong mục này, chúng tôi trình bày cách mô hình hóa sự dao động kênh tương quan thời gian khi thiết bị đầu cuối di động di chuyển. Hơn nữa, chúng tôi trình bày một số phương pháp thực tế để triển khai mô hình kênh di động ngoài trời cho cả kênh truyền phẳng và kênh chọn lọc tần số.

2.2.1 Mô hình FWGN

Kênh ngoài trời được đặc trưng bởi phổ Doppler chi phối sự biến đổi theo thời gian của độ lợi kênh. Các loại phổ Doppler khác nhau có thể được tạo ra bằng mô hình nhiễu Gauss trắng được lọc (FWGN). Mô hình FWGN là một trong những mô hình kênh ngoài trời phổ biến nhất. Mô hình Clarke/Gans là mô hình FWGN cơ bản có thể được điều chỉnh thành các loại khác nhau, tùy thuộc vào cách thức bộ lọc Doppler được triển khai trong miền thời gian hoặc miền tần số. Trước hết chúng ta thảo luận về mô hình Clarke/Gans và sau đó là các biến thể trong miền tần số và miền thời gian của nó.

2.2.1.1 Mô hình Clarke/Gans

Mô hình Clarke/Gans được xây dựng với giả thuyết là các thành phần tán xạ quanh trạm di động được phân bố đồng đều trong đó công suất cho mỗi thành phần bằng nhau [26]. Hình 2.6 biểu diễn sơ đồ khối của mô hình Clarke/Gans, chúng ta thấy có hai nhánh, một nhánh cho phần thực và nhánh còn lại ứng với phần ảo. Trong mỗi nhánh, trước hết nhiễu Gauss phức được tạo ra trong miền tần số và sau đó được lọc bằng bộ lọc Doppler để thành phần tần số chịu dịch chuyển Doppler. Cuối cùng, nhiễu Gauss dịch chuyển Doppler được chuyển thành tín hiệu trong miền thời gian thông qua khối IFFT. Bởi vì đầu ra của khối IFFT phải là tín hiệu thực, đầu vào của nó phải luôn luôn đối xứng liên hợp. Xây dựng độ lợi kênh phức bằng cách thêm phần thực vào phần ảo của đầu ra, kênh có biên độ phân bố Rayleigh được hình thành.



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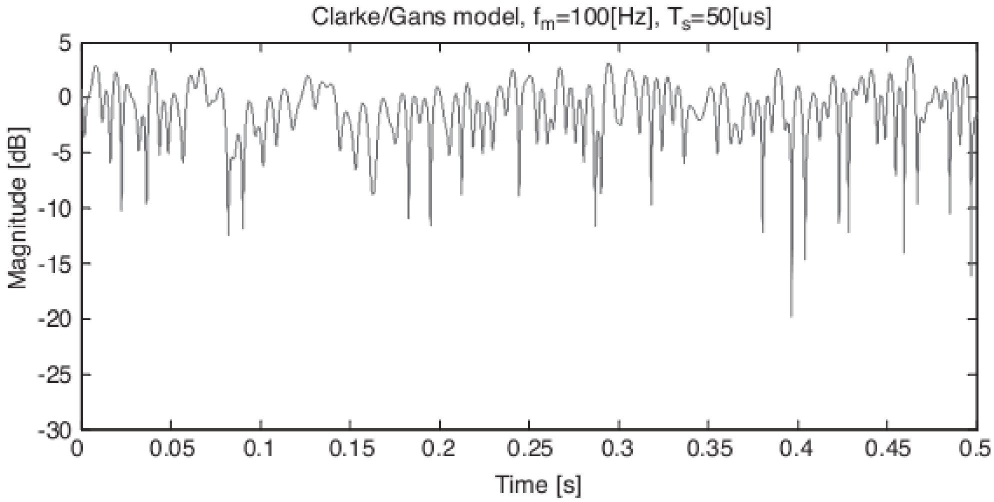
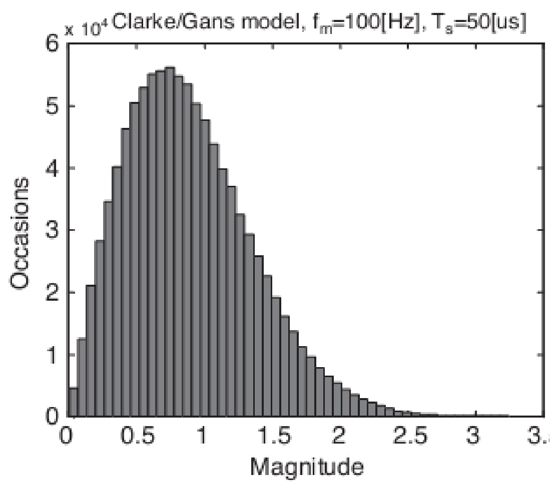
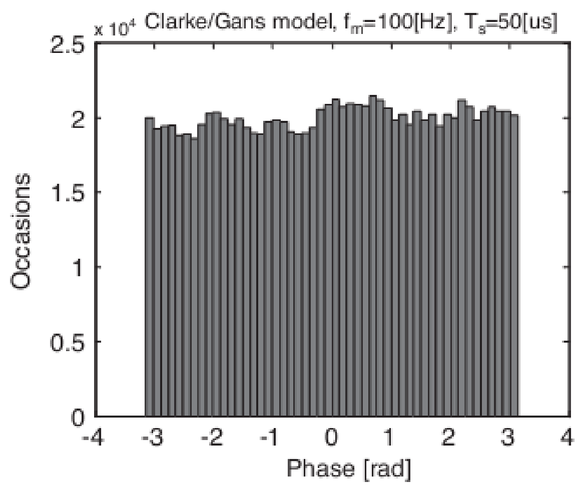


(a) Time-domain channel characteristics



(b) Distribution of magnitude



(c) Distribution of phase

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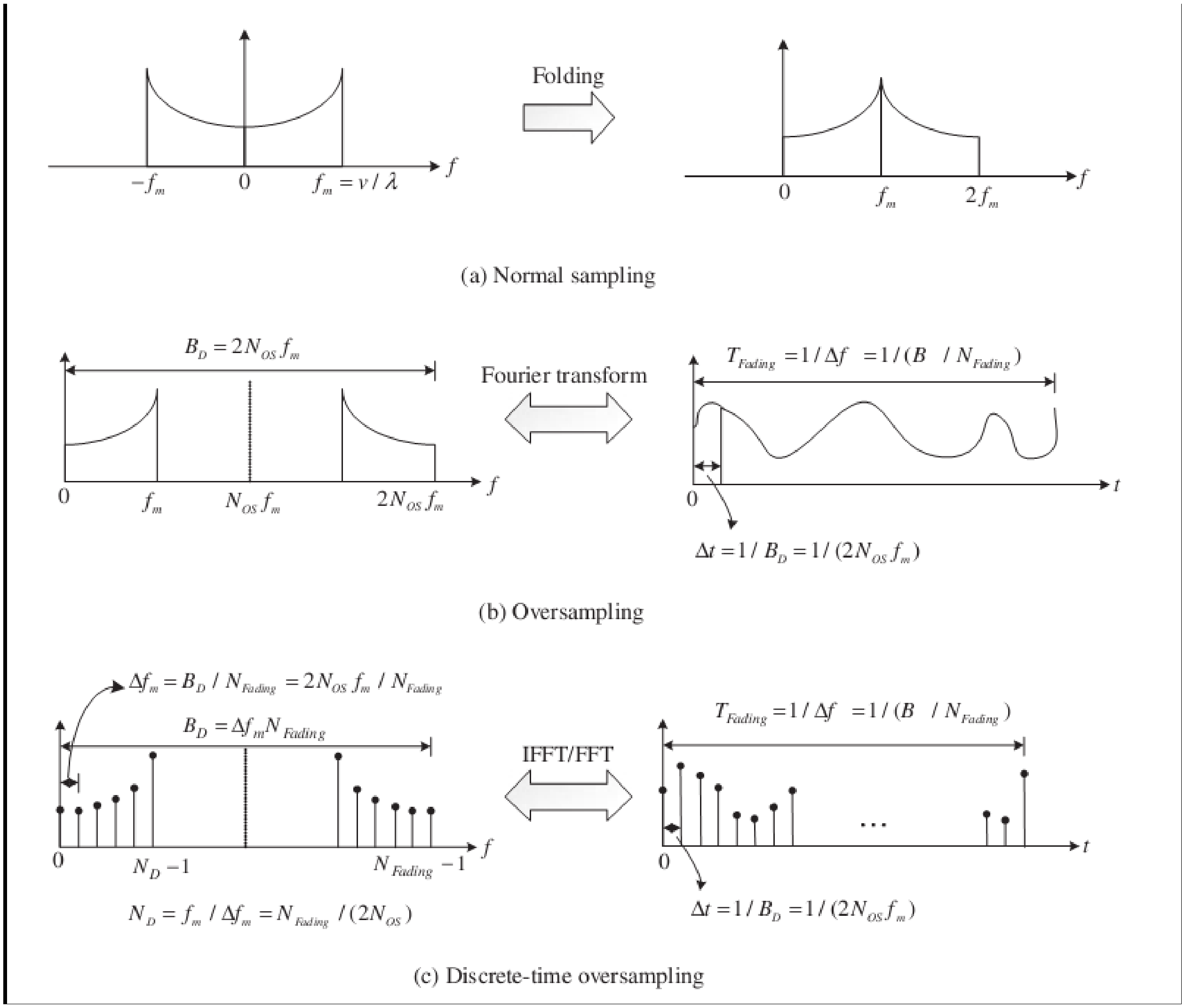

(a) Normal sampling

(b) Oversampling

(c) Discrete-time oversampling

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$$h[n] = \sum_{k=-N_{Fading}/2}^{N_{Fading}/2-1} \sqrt{S[k]} e^{j\theta_k} e^{j2\pi nk/N_{Fading}} \qquad (2.20)$$

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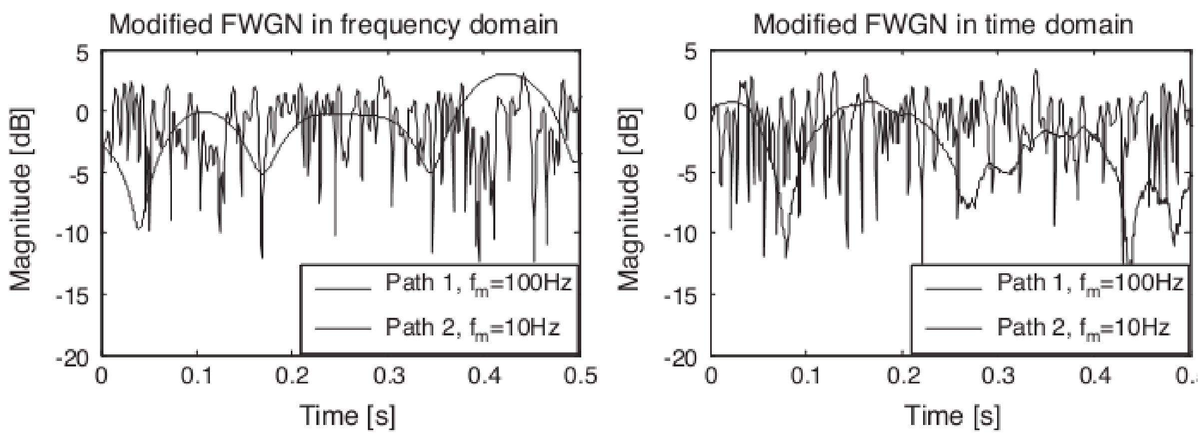

(a) Modified frequency-domain FWGN model (b) Modified time-domain FWGN model

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$$S(f) \propto 1, \quad |f| \le f_m \tag{2.21}$$

[redacted]

$$S(f) \propto \frac{1}{\sqrt{1-(f/f_m)^2}} \cdot \left\{ \exp\left(-\frac{\sqrt{2}}{\sigma}\left|\cos^{-1}(f/f_m)-\phi\right|\right) + \exp\left(-\frac{\sqrt{2}}{\sigma}\left|\cos^{-1}(f/f_m)+\phi\right|\right) \right\}, |f| \le f_m \quad (2.22)$$

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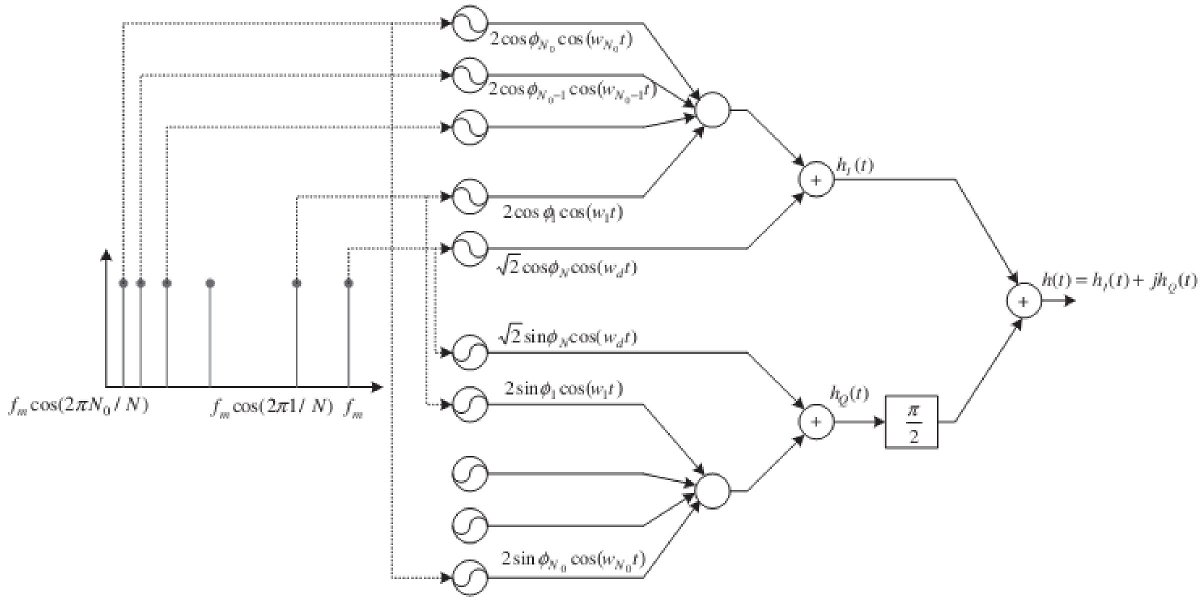

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$$h_I(t) = 2\sum_{n=1}^{N_0}(\cos\phi_n \cos w_n t) + \sqrt{2}\cos\phi_N \cos w_d t \qquad (2.23a)$$

[redacted]

$$h_Q(t) = 2\sum_{n=1}^{N_0}(\sin\phi_n \cos w_n t) + \sqrt{2}\sin\phi_N \cos w_d t \qquad (2.23b)$$

[redacted]

$$\begin{aligned}\phi_N &= 0 \\ \phi_n &= \pi n/(N_0+1), \qquad n = 1,2,\cdots,N_0\end{aligned} \tag{2.24}$$

[redacted]

$$h(t) = \frac{E_0}{\sqrt{2N_0+1}}\left\{h_I(t) + jh_Q(t)\right\} \tag{2.25}$$

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$$w_n = w_d \cos\theta_n = 2\pi f_m \cos(2\pi n/N), \qquad n = 1,2,\cdots,N_0 \tag{2.26}$$

[redacted]

$$E\left\{\left(\frac{E_0 h_I(t)}{\sqrt{2N_0+1}}\right)^2\right\} = E\left\{\left(\frac{E_0 h_Q(t)}{\sqrt{2N_0+1}}\right)^2\right\} = \frac{E_0^2}{2} \tag{2.27}$$

$$E\left\{h^2(t)\right\} = E_0^2 \tag{2.28}$$

$$E\{h(t)\} = E_0 \tag{2.29}$$

$$E\left\{h_I(t)h_Q(t)\right\} = 0 \tag{2.30}$$

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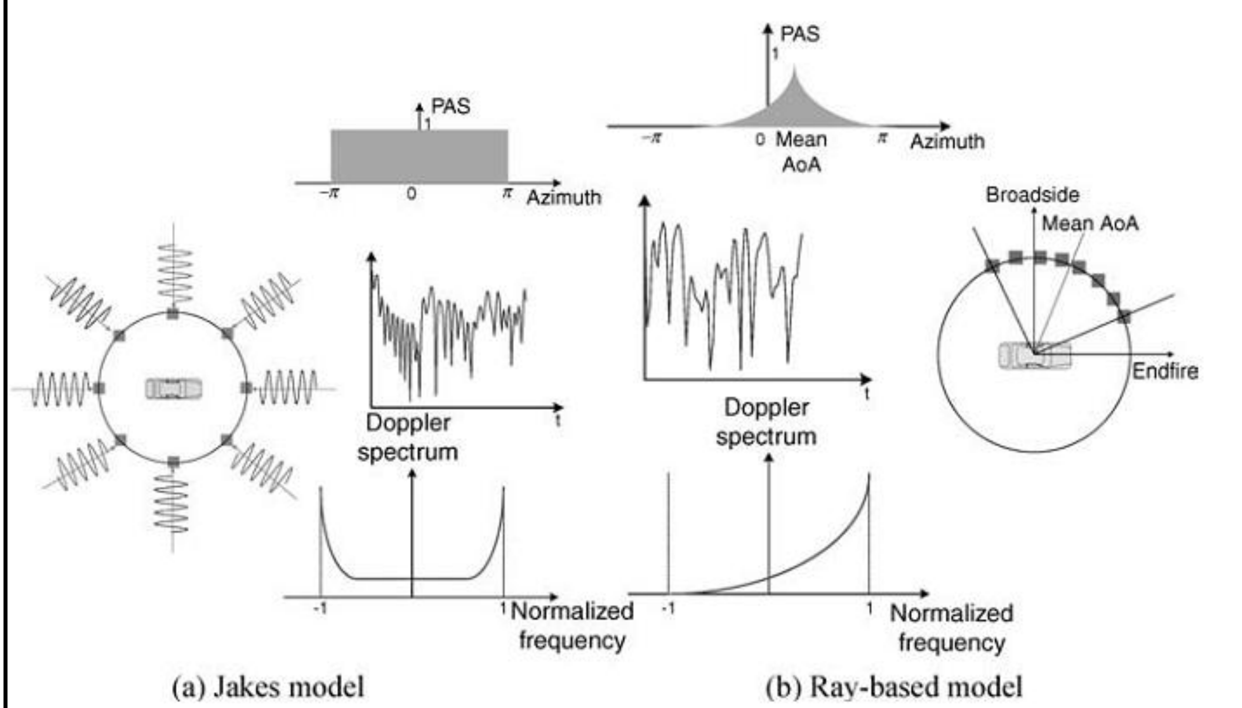

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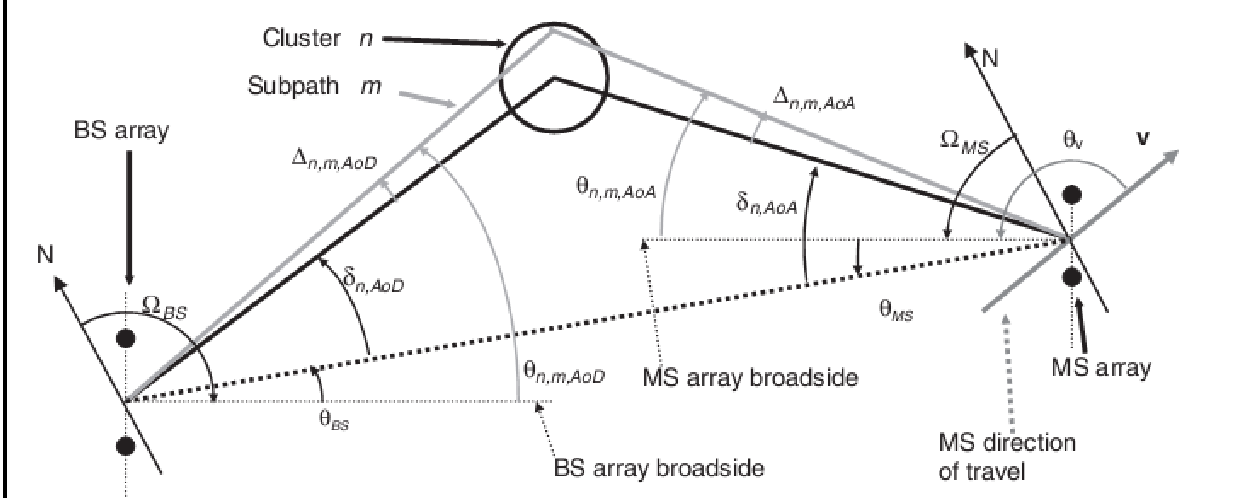

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$$h_{u,s,n}(t)=\sqrt{\frac{P_n\sigma_{SF}}{M}}\sum_{m=1}^{M}\left(\begin{array}{l}\sqrt{G_{BS}(\theta_{n,m,AoD})}\exp\left(j\left[kd_s\sin(\theta_{n,m,AoD})+\Phi_{n,m}\right]\right)\times\\ \sqrt{G_{MS}(\theta_{n,m,AoA})}\exp\left(jkd_u\sin\theta_{n,m,AoA}\right)\exp\left(jk\|\mathbf{v}\|\cos(\theta_{n,m,AoA}-\theta_v)t\right)\end{array}\right) \quad (2.31)$$

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$$h_n(t) = \sqrt{\frac{P_n}{M}} \sum_{m=1}^{M} \left(\exp(j\Phi_{n,m}) \times \exp\left(j\frac{2\pi}{\lambda} ||\mathbf{v}|| \cos\left(\theta_{n,m,AoA} - \theta_v\right) t \right) \right) \qquad (2.32)$$

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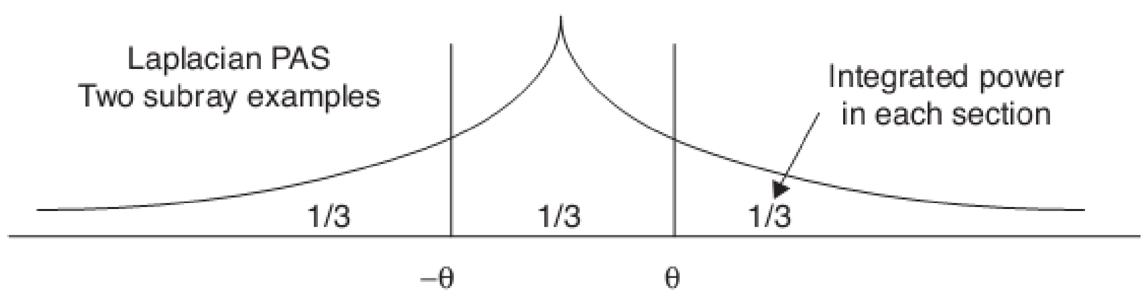

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$$\int_{\theta_1}^{\theta_2} P(\theta,\sigma)d\theta = \int_{\theta_1}^{\theta_2} \frac{1}{\sqrt{2}\sigma} e^{\frac{-\sqrt{2}|\theta|}{\sigma}} d\theta$$
$$= -\frac{1}{2}\left(e^{\frac{-\sqrt{2}|\theta_2|}{\sigma}} - e^{\frac{-\sqrt{2}|\theta_1|}{\sigma}}\right)$$
$$= \frac{1}{a(M+1)} \tag{2.33}$$

[redacted]

$$\int_0^{\theta} P(\theta,\sigma)d\theta = 1/6 \text{ such that } a = 2.$$

[redacted]

$$\theta_{m+1}[\text{deg}] = -\frac{\sigma}{\sqrt{2}}\left[\ln\left(e^{\frac{-\sqrt{2}\theta_m}{\sigma}} - \frac{2}{a(M+1)}\right)\right],$$
$$m = 0, 1, 2, \cdots, \lfloor M/2 \rfloor - 1 \text{ and } \theta_0 = 0^\circ \tag{2.34}$$

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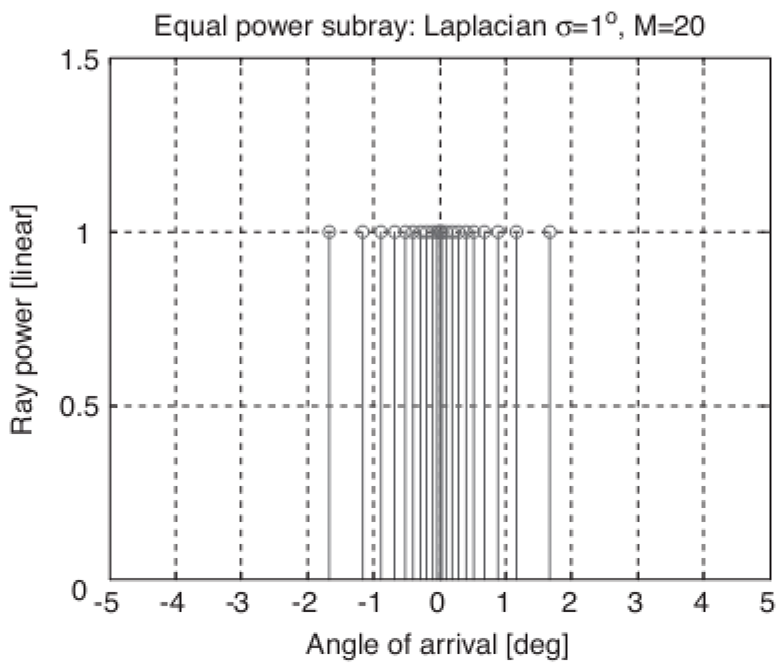

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Sub-path # (m)	Offset for a 2 deg AS at BS (Macrocell) (degrees)	Offset for a 5 deg AS at BS (Microcell) (degrees)	Offset for a 35 deg AS at MS (degrees)
1, 2	±0.0894	± 0.2236	±1.5649
3, 4	±0.2826	± 0.7064	±4.9447
5, 6	±0.4984	±1.2461	±8.7224
7, 8	±0.7431	±1.8578	±13.0045
9, 10	±1.0257	± 2.5642	±17.9492
11, 12	±1.3594	± 3.3986	± 23.7899
13, 14	±1.7688	± 4.4220	± 30.9538
15, 16	±2.2961	±5.7403	±40.1824
17, 18	±3.0389	±7.5974	±53.1816
19, 20	±4.3101	±10.7753	±75.4274

[redacted]

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$$\theta_m = -\alpha + m \cdot \delta + \phi \quad \text{for} \quad m = 0, 1, \cdots, M-1 \qquad (2.35)$$

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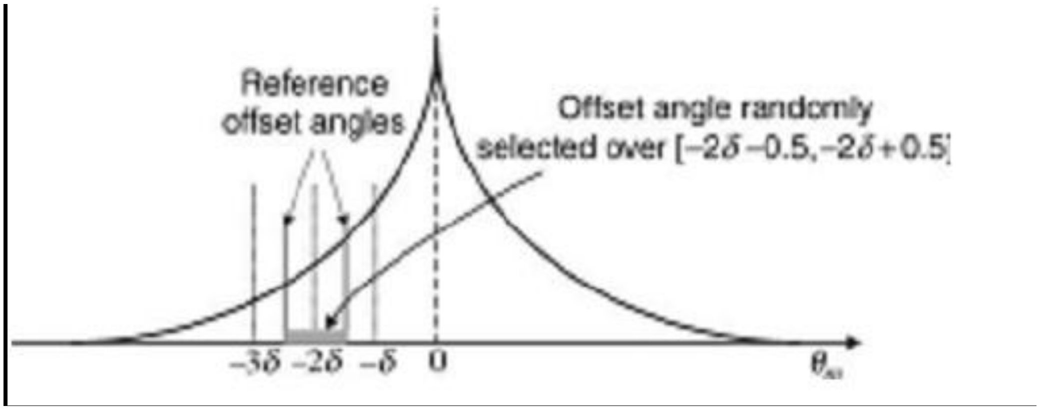

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Tab	Pedestrian A		Pedestrian B		Vehicular A		Vehicular B		Doppler spectrum
	Relative delay [ns]	Average power [dB]	Relative delay [ns]	Average power [dB]	Relative delay [ns]	Average power [dB]	Relative delay [ns]	Average power [dB]	
1	0	0.0	0.	0.0	0	0.0	0	−2.5	Classic
2	110	−9.7	200	−0.9	310	−1.0	300	0.0	Classic
3	190	−19.2	800	−4.9	710	−9.0	8900	−12.8	Classic
4	410	−22.8	1200	−8.0	1090	−10.0	12 900	−10.0	Classic
5			2300	−7.8	1730	−15.0	17 100	−25.2	Classic
6			3700	−23.9	2510	−20.0	20 000	−16.0	Classic

[redacted]

Tab	Typical urban (TU)			Bad urban (BU)		
	Relative delay [us]	Average power	Doppler spectrum	Relative delay [us]	Average power	Doppler spectrum
1	0.0	0.189	Classic	0.0	0.164	Classic
2	0.2	0.379	Classic	0.3	0.293	Classic
3	0.5	0.239	Classic	1.0	0.147	GAUS1
4	1.6	0.095	GAUS1	1.6	0.094	GAUS1
5	2.3	0.061	GAUS2	5.0	0.185	GAUS2
6	5.0	0.037	GAUS2	6.6	0.117	GAUS2

[redacted]

Tab	Typical urban (TU)			Bad urban (BU)		
	Relative delay [us]	Average power	Doppler spectrum	Relative delay [us]	Average power	Doppler spectrum
1	0.0	0.092	Classic	0.0	0.033	Classic
2	0.1	0.115	Classic	0.1	0.089	Classic
3	0.3	0.231	Classic	0.3	0.141	Classic
4	0.5	0.127	Classic	0.7	0.194	GAUS1
5	0.8	0.115	GAUS1	1.6	0.114	GAUS1
6	1.1	0.074	GAUS1	2.2	0.052	GAUS2
7	1.3	0.046	GAUS1	3.1	0.035	GAUS2
8	1.7	0.074	GAUS1	5.0	0.140	GAUS2
9	2.3	0.051	GAUS2	6.0	0.136	GAUS2
10	3.1	0.032	GAUS2	7.2	0.041	GAUS2
11	3.2	0.018	GAUS2	8.1	0.019	GAUS2
12	5.0	0.025	GAUS2	10.0	0.006	GAUS2

[redacted]

Tab	Typical rural area (RA)		
	Relative delay [us]	Average power	Doppler spectrum
1	0.0	0.602	RICE
2	0.1	0.241	Classic
3	0.2	0.096	Classic
4	0.3	0.036	Classic
5	0.4	0.018	Classic
6	0.5	0.006	Classic

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$$y(n) = \sum_{d=0}^{N_D-1} h_d(n)x(n-d) \quad (2.36)$$

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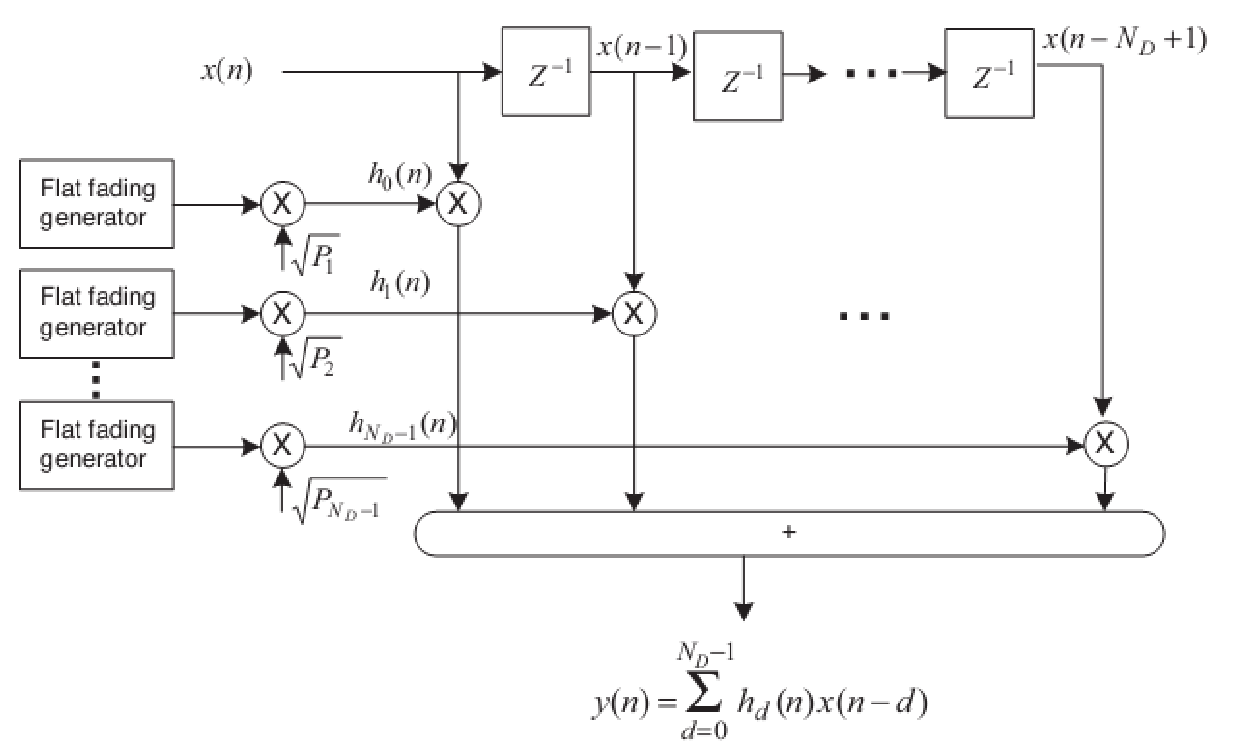

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$$t'_d = floor(t_d/t_s + 0.5) \cdot t_s \qquad (2.37)$$

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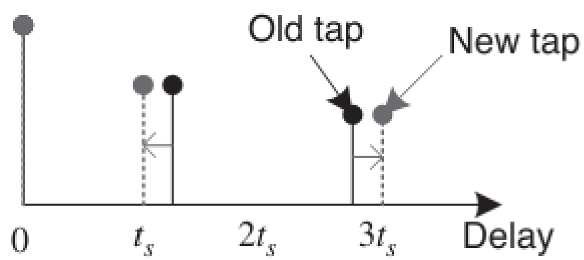

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$$t_r = t_d/t_s - t_i \qquad (2.38)$$

[redacted]

$$h'_{t_i}(n) = \tilde{h}_{t_i}(n) + \sqrt{1-t_r}\,h_{t_d}(n) \quad (2.39)$$

[redacted]

$$\tilde{h}_{t_i+1}(n) = \sqrt{t_r}\,h_{t_d}(n) \quad (2.40)$$

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Terrain type	SUI channels
A	SUI-5, SUI-6
B	SUI-3, SUI-4
C	SUI-1, SUI-2

[redacted]

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$$S(f) = \begin{cases} 1-1.72 f_0^2 + 0.785 f_0^4 & f_0 \le 1 \\ 0 & f_0 > 1 \end{cases} \quad (2.41)$$

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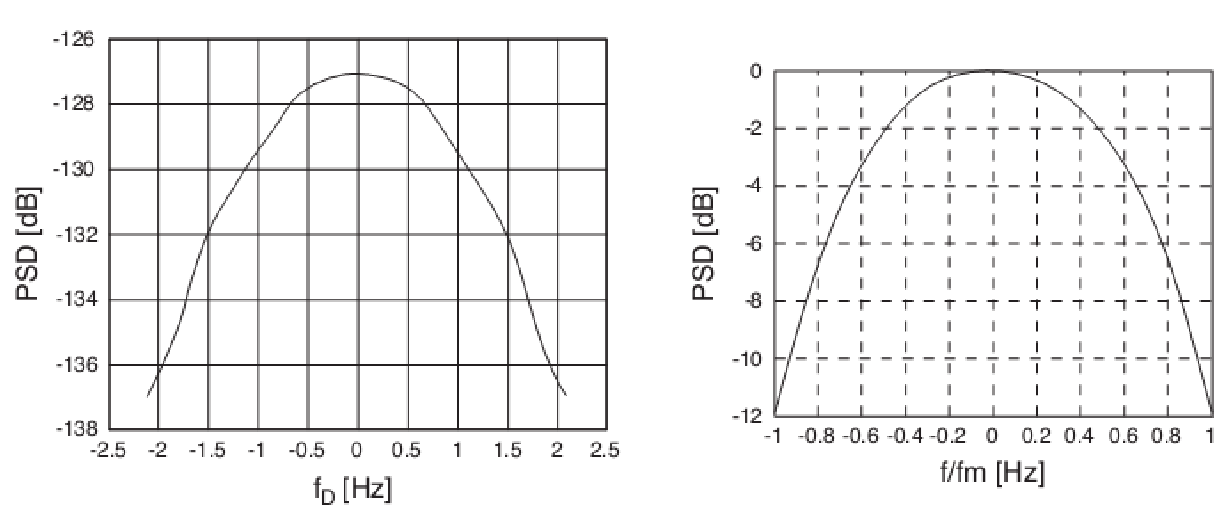

(a) Measurement of Doppler PSD @ 2.5GHz

(b) Truncated Doppler PSD

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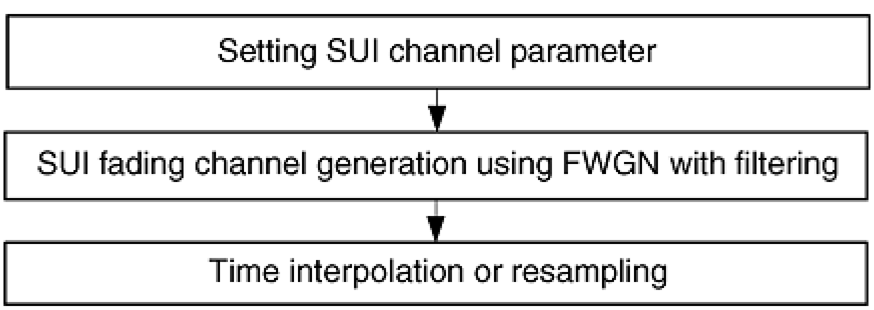
SUI 1/2/3/4/5/6 channel			
	Tap 1	Tap 2	Tap 3
Delay [μs]	0/0/0/0/0/0	0.4/0.4/0.4/1.5/4/14	0.9/1.1/0.9/4/10/20
Power (omni ant.) [dB]	0/0/0/0/0/0	-15/-12/-5/-4/-5/-10	-20/-15/-10/-8/-10/-14
90% K-factor (omni)	4/2/1/0/0/0	0/0/0/0/0/0	0/0/0/0/0/0
75% K-factor (omni)	20/11/7/1/0/0	0/0/0/0/0/0	0/0/0/0/0/0
50% K-factor (omni)	-/-/-/-/2/1	-/-/-/-/0/0	-/-/-/-/0/0
Power (30° ant.) [dB]	0/0/0/0/0/0	-21/-18/-11/-10/-11/-16	-32/-27/-22/-20/-22/-26
90% K-factor (30° ant.)	16/8/3/1/0/0	0/0/0/0/0/0	0/0/0/0/0/0
75% K-factor (30° ant.)	72/36/19/5/2	0/0/0/0/0/0	0/0/0/0/0/0
50% K-factor (30° ant.)	-/-/-/-/7/5	-/-/-/-/0/0	-/-/-/-/0/0
Doppler [Hz]	0.4/0.2/0.4/0.2/2/0.4	0.3/0.15/0.3/0.15/1.5/0.3	0.5/0.25/0.5/0.25/2.5/0.5
Antenna correlation	$\rho_{ENV} = 0.7/0.5/0.4/0.3/0.5/0.3$		
Gain reduction factor	$G_{RF} = 0/2/3/4/4/4\,dB$		
Normalization factor	$F_{omni} = -0.1771/-0.3930/-1.5113/-1.9218/-1.5113/-0.5683dB$ $F_{30^\circ} = -0.0371/-0.0768/-0.3573/-0.4532/-0.3573/-0.1184dB$		
Terrain type	C/C/B/B/A/A		
Omni antenna: overall K	$\sigma_\tau = 0.111/0.202/0.264/1.257/2.842/5.240\,\mu s$ K=3.3/16/0.5/0.2/0.1/0.1(90%) K = 10.4/5.1/1.6/0.6/0.3/0.3 (75%), K = -/-/-/-/1.0/1.0 (50%)		
30° antenna: overall K	$\sigma_\tau = 0.042/0.69/0.123/0.563/1.276/2.370\,\mu s$ K = 14.0/6.9/2.2/1.0/0.4/0.4 (90%), K = 44.2/21.8/7.0/3.2/1.3/1.3 (75%), K = -/-/-/-/4.2/4.2 (50%)		



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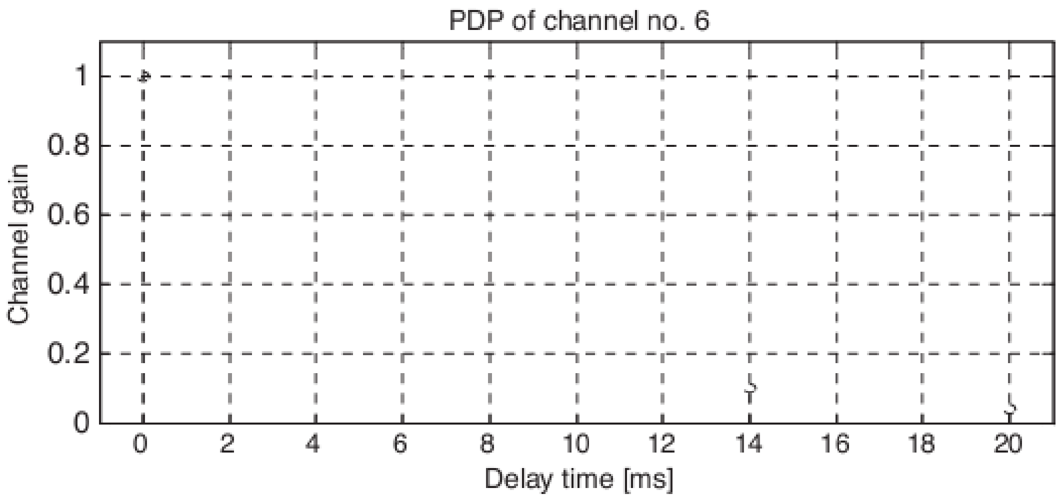
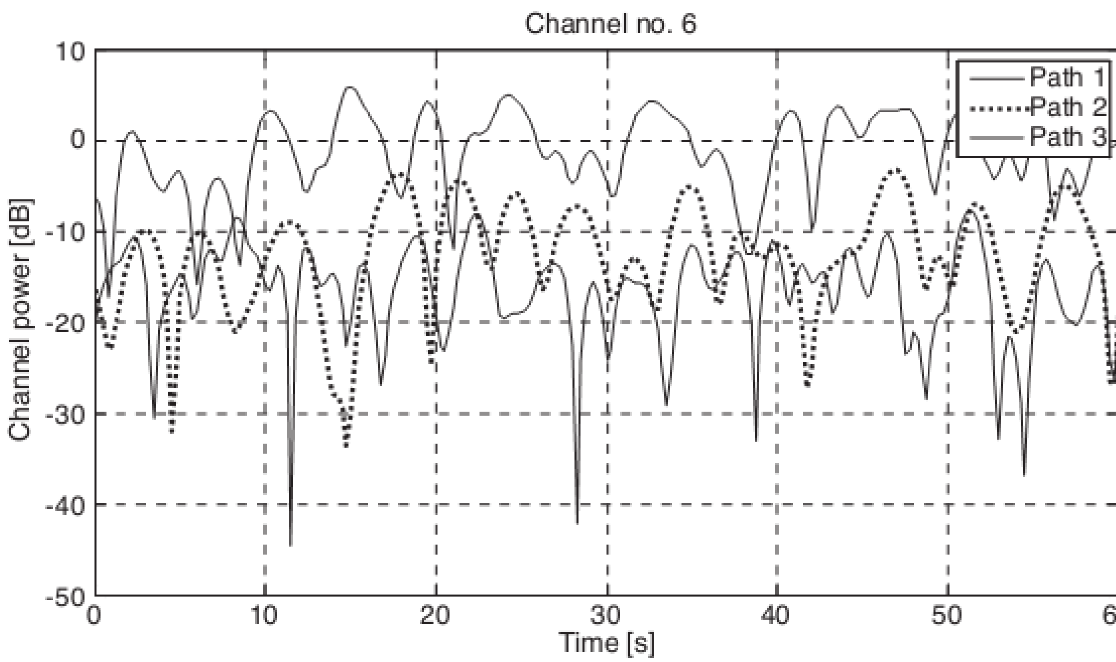


(a) Power delay profile (PDP)



(b) Time-domain channel characteristic

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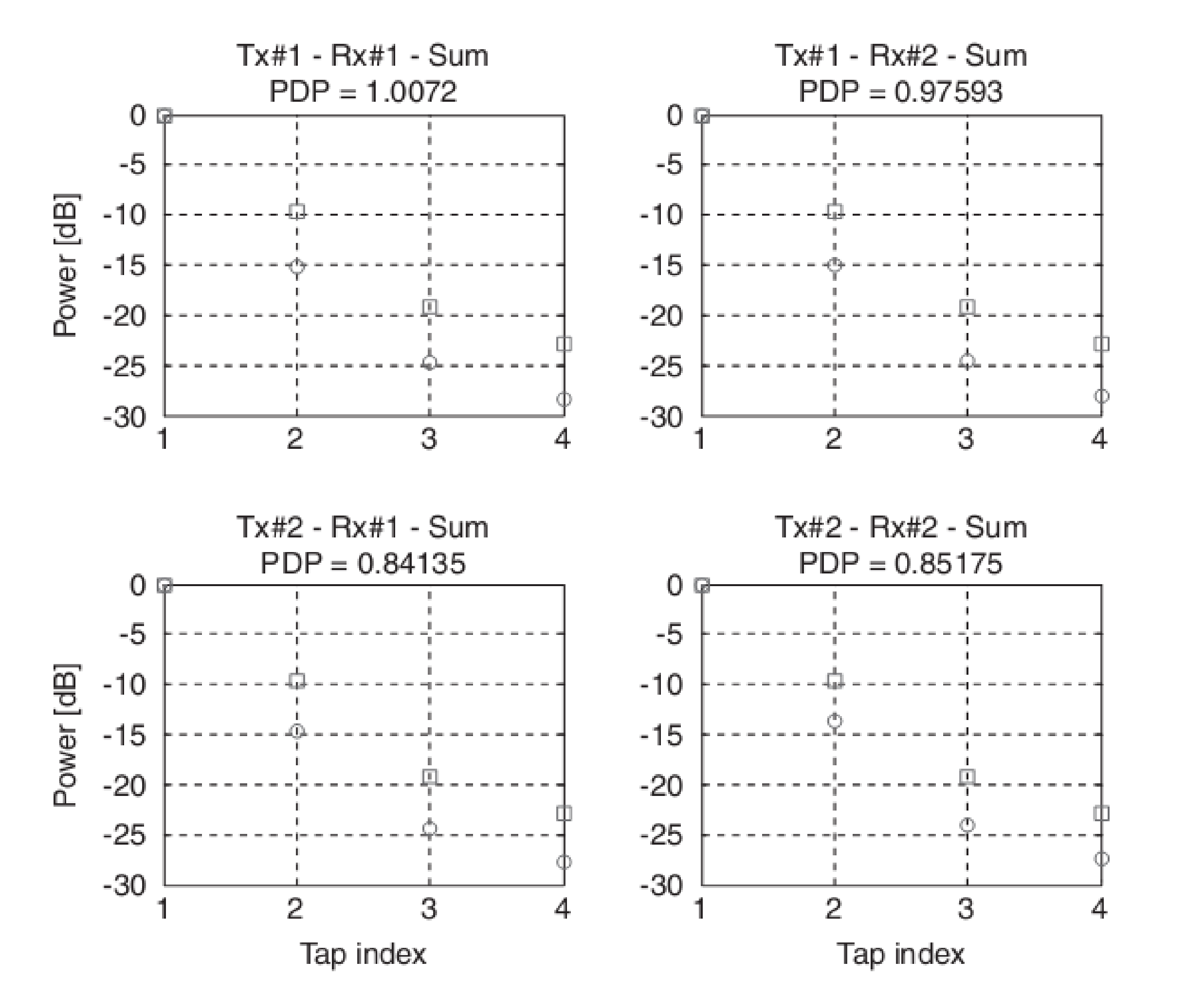

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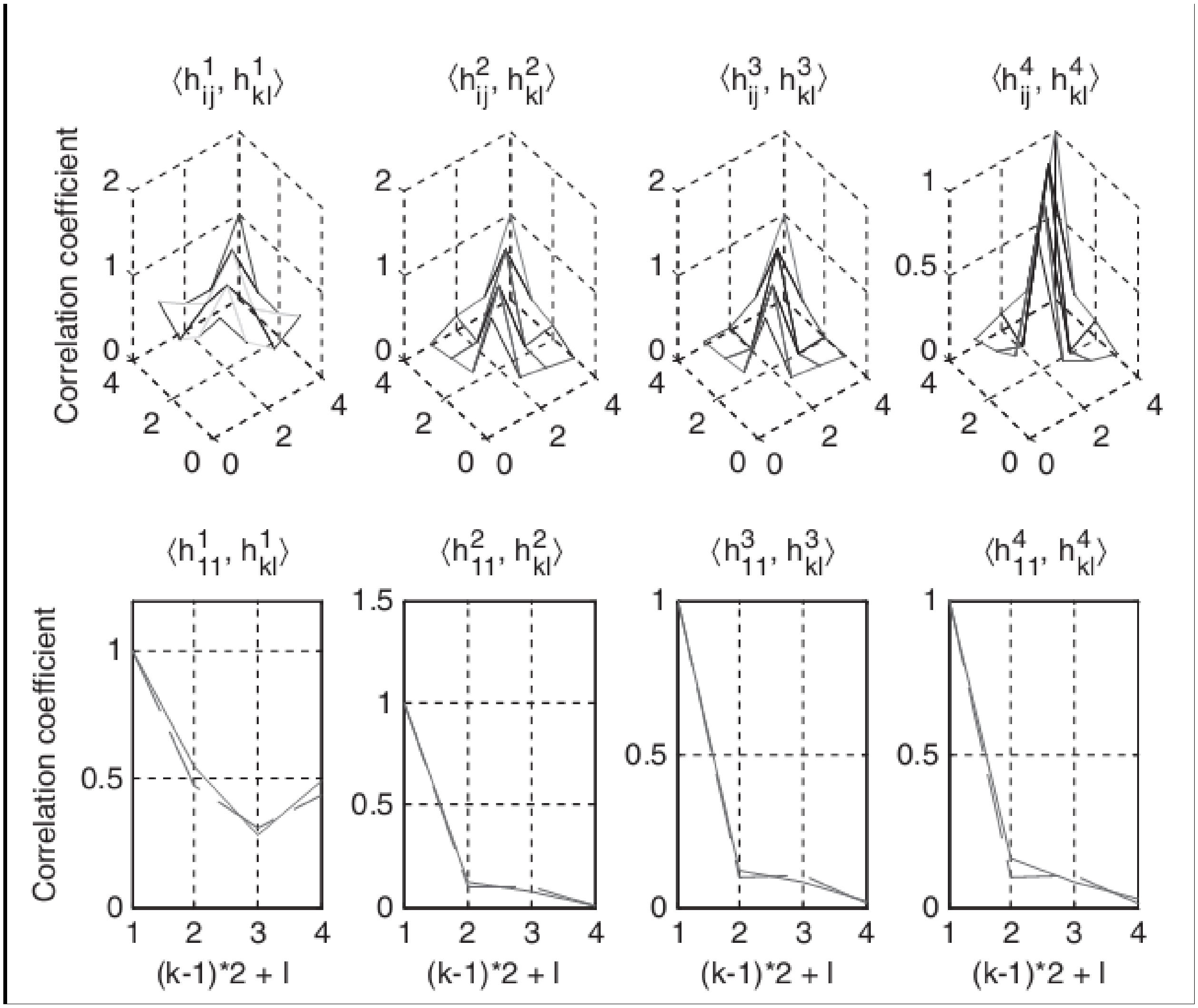

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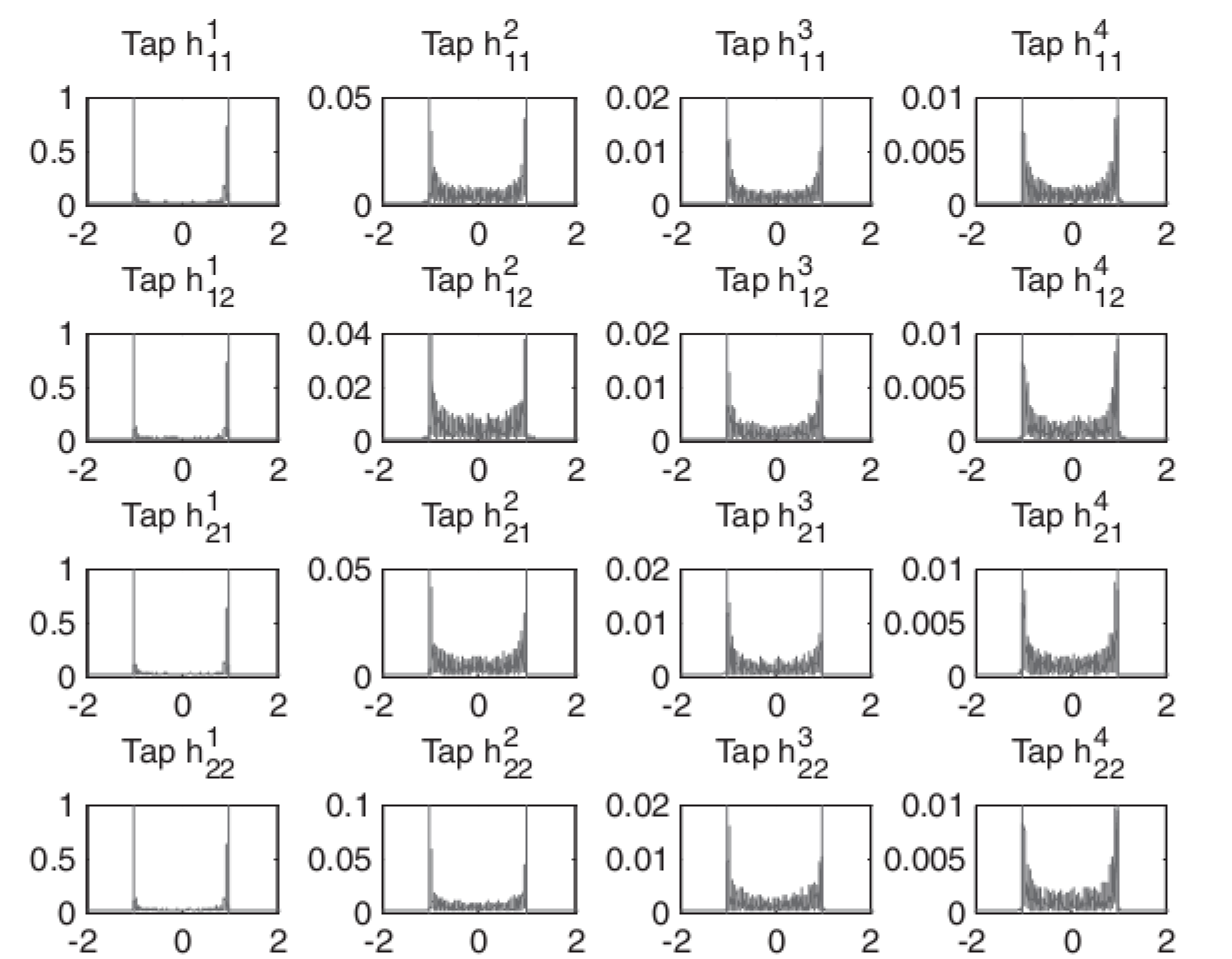
$$P(\theta, \sigma, \bar{\theta}) = N_o e^{\frac{-\sqrt{2}|\theta-\bar{\theta}|}{\sigma}} G(\theta) \tag{3.63}$$

[redacted]

$$\frac{1}{N_0} = \int_{-\pi+\bar{\theta}}^{\pi+\bar{\theta}} e^{\frac{-\sqrt{2}|\theta-\bar{\theta}|}{\sigma}} G(\theta) d\theta, -\pi + \bar{\theta} \le \theta \le \pi + \bar{\theta} \tag{3.64}$$

[redacted]

$$A(\theta) = -\min\left[12\left(\frac{\theta}{\theta_{3\text{ dB}}}\right)^2, A_m\right] \text{[dB]}, \quad \text{for} \quad -180^\circ \leq \theta \leq -180^\circ \qquad (3.65)$$



[redacted]

[redacted]

[redacted]

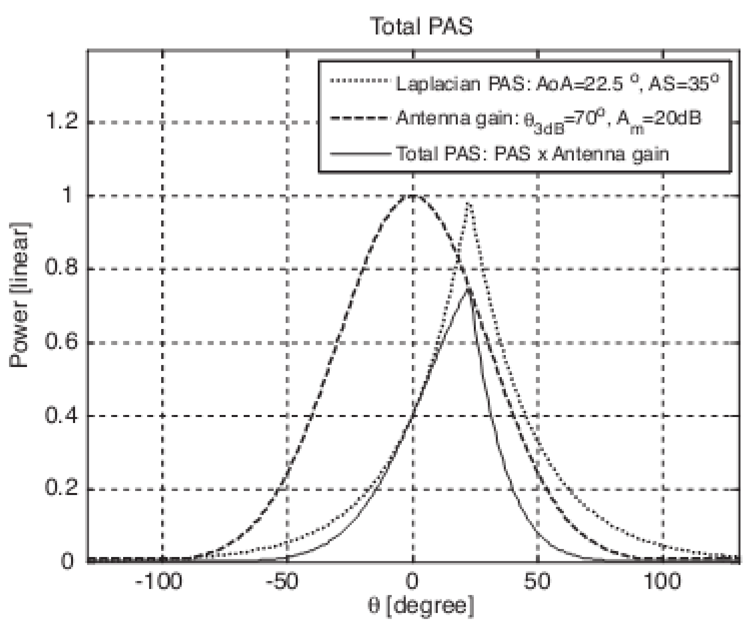
[redacted]

$$P(\theta,\sigma,\bar{\theta}) = N_o e^{\frac{-\sqrt{2}|\theta-\bar{\theta}|}{\sigma}}, \qquad -\pi+\bar{\theta} \le \theta \le \pi+\bar{\theta} \tag{3.66}$$

[redacted]

$$\frac{1}{N_0} = \int_{-\pi+\bar{\theta}}^{\pi+\bar{\theta}} e^{\frac{-\sqrt{2}|\theta-\bar{\theta}|}{\sigma}} d\theta \ = \sqrt{2}\sigma\left(1-e^{-\sqrt{2}\pi/\sigma}\right) \tag{3.67}$$

[redacted]



[redacted]

[redacted]

$$P(\theta, \sigma, \bar{\theta}) = N_0 \cdot 1, \qquad -\sqrt{3}\sigma + \bar{\theta} \leq \theta \leq \sqrt{3}\sigma + \bar{\theta} \tag{3.68}$$

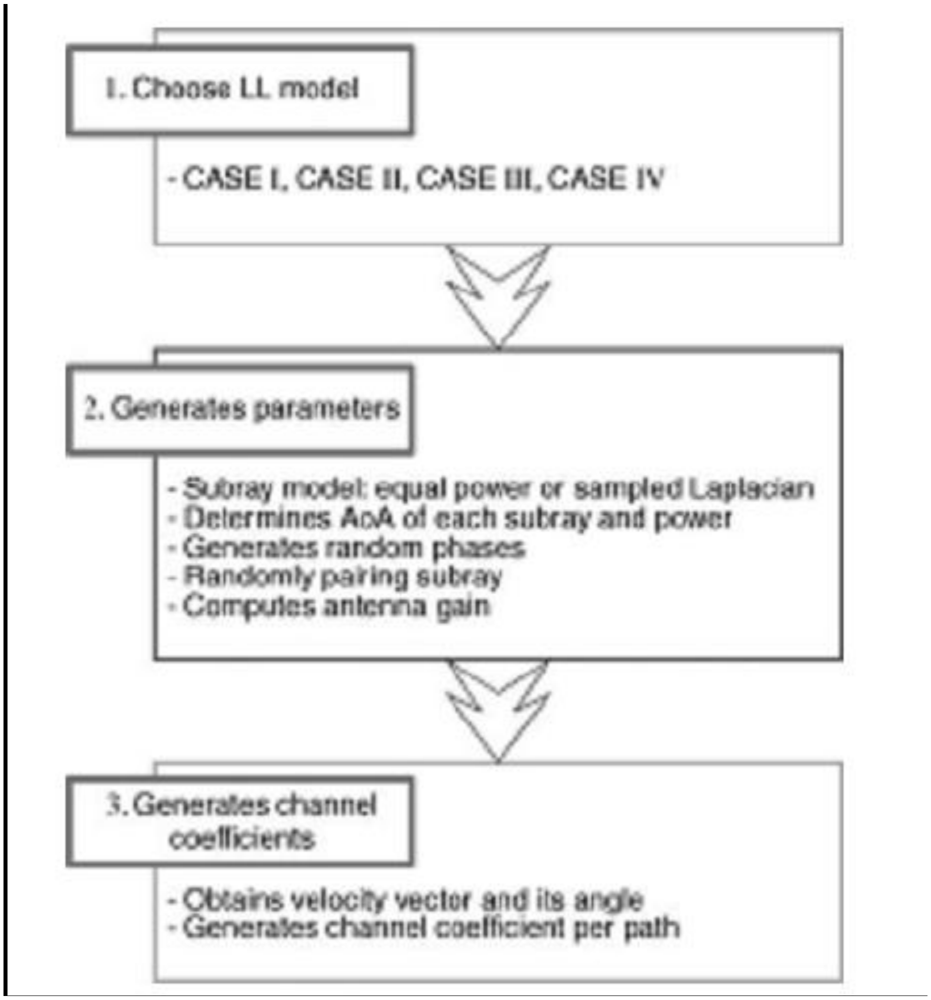
[redacted]

[redacted]

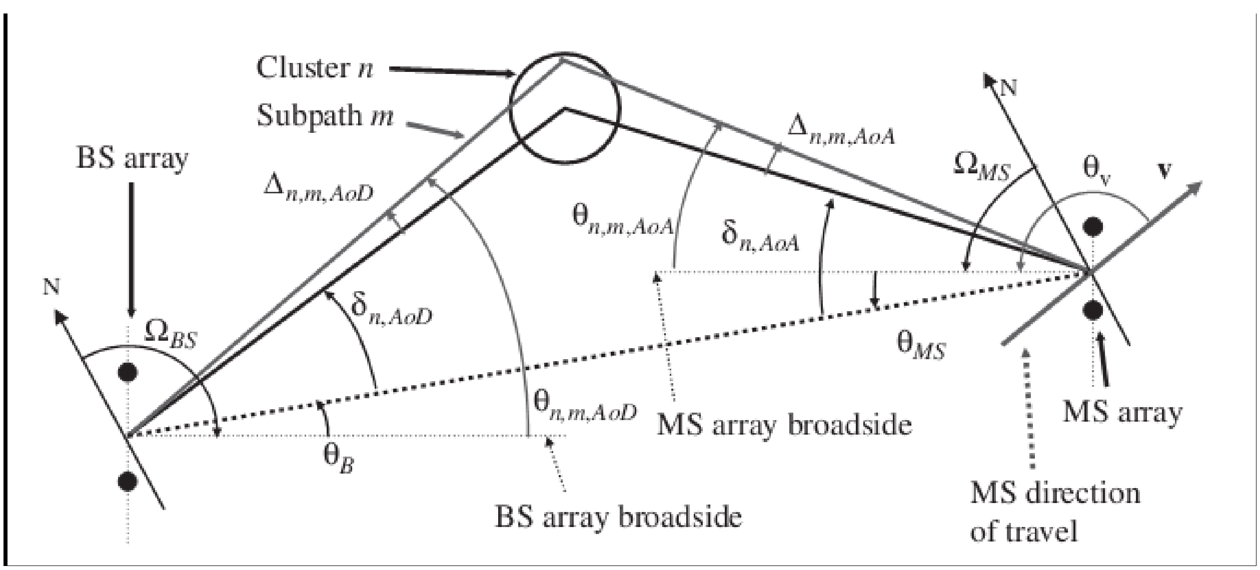
[redacted]

[redacted]

$$h_{s,u,n}(t) = \sqrt{\begin{matrix} n\text{th Path} \\ \text{Power} \end{matrix}} \sum_{m=1}^{M} \left\{ \begin{pmatrix} \text{BS} \\ \text{PAS} \end{pmatrix} \cdot \begin{pmatrix} \text{Phase due to} \\ \text{BS Array} \end{pmatrix} \cdot \begin{pmatrix} \text{MS} \\ \text{PAS} \end{pmatrix} \cdot \begin{pmatrix} \text{Phase due to} \\ \text{MS Array} \end{pmatrix} \right\} \tag{3.69}$$


1. Choose LL model
- CASE I, CASE II, CASE III, CASE IV
2. Generates parameters
- Subray model: equal power or sampled Laplacian
- Determines AoA of each subray and power
- Generates random phases
- Randomly pairing subray
- Computes antenna gain
3. Generates channel coefficients
- Obtains velocity vector and its angle
- Generates channel coefficient per path

[redacted]


Cluster n
Subpath m
BS array
N
Ω_{BS}
$\Delta_{n,m,AoD}$
$\delta_{n,AoD}$
$\theta_{n,m,AoD}$
θ_B
BS array broadside
$\Delta_{n,m,AoA}$
$\theta_{n,m,AoA}$
$\delta_{n,AoA}$
MS array broadside
θ_{MS}
Ω_{MS}
θ_v
v
N
MS array
MS direction
of travel

[redacted]

[redacted]

$$h_{u,s,n}(t) = \sqrt{\frac{P_n}{M}} \sum_{m=1}^{M} \begin{pmatrix} \sqrt{G_{BS}(\theta_{n,m,AoD})} \exp\left(j\left[kd_s \sin(\theta_{n,m,AoD}) + \Phi_{n,m}\right]\right) \times \\ \sqrt{G_{MS}(\theta_{n,m,AoA})} \exp\left(jkd_u \sin(\theta_{n,m,AoA})\right) \times \\ \exp\left(jk\|\mathbf{v}\|\cos(\theta_{n,m,AoA} - \theta_v)\, t\right) \end{pmatrix} \tag{3.70}$$

[redacted]

[redacted]

$$h_{u,s,n}(t) = \sqrt{P_n} \sum_{m=1}^{M} \begin{pmatrix} \sqrt{P_{BS}(\theta_{n,m,AoD}) G_{BS}(\theta_{n,m,AoD})} \exp\left(j\left[kd_s \sin\theta_{n,m,AoD} + \Phi_{n,m}\right]\right) \times \\ \sqrt{P_{MS}(\theta_{n,m,AoD}) G_{MS}(\theta_{n,m,AoA})} \exp\left(jkd_u \sin\theta_{n,m,AoA}\right) \\ \times \exp\left(jk\|\mathbf{v}\|\cos(\theta_{n,m,AoA} - \theta_v)\, t\right) \end{pmatrix} \tag{3.71}$$

[redacted]

$$h_{s,u,n=1}^{LOS}(t) = \sqrt{\frac{1}{K+1}}h_{s,u,1}(t) + \sqrt{\frac{K}{K+1}}\begin{pmatrix} \sqrt{G_{BS}(\theta_{BS})}\exp(jkd_s \sin\theta_{BS})\times \\ \sqrt{G_{MS}(\theta_{MS})}\exp(jkd_u \sin\theta_{MS} + \Phi_{LOS})\times \\ \exp(jk\|\mathbf{v}\|\cos(\theta_{MS}-\theta_v)t) \end{pmatrix} \quad (3.72)$$

[redacted]

$$h_{s,u,n}^{LOS}(t) = \sqrt{\frac{1}{K+1}}h_{s,u,n}(t), \quad \text{for} \quad n \neq 1 \quad (3.73)$$

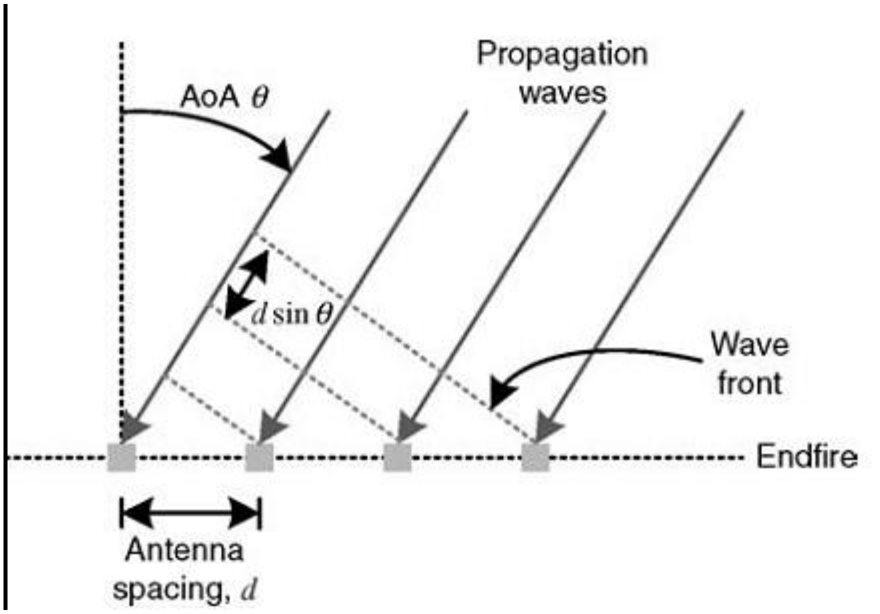
[redacted]

$$\rho(d) = E\left\{h_{1,u,n}(t) \cdot h_{2,u,n}^{*}(t)\right\} = \int_{-\pi}^{\pi} e^{\frac{j2\pi d \ \sin\theta}{\lambda}}P(\theta)d\theta \quad (3.74)$$

[redacted]

$$\rho_{SCM}^{sc}(d) = \frac{1}{M}\sum_{m=1}^{M} e^{\frac{j2\pi d\ \sin\theta_{n,m,AoA}}{\lambda}} \tag{3.75}$$

[redacted]



[redacted]

[redacted]

$$
\begin{aligned}
\rho_{SCM}^{tc}(\tau) &= E\left\{h_{s,u,n}(t+\tau)\cdot h_{s,u,n}^{*}(t)\right\} \\
&= \frac{1}{M}\sum_{m=1}^{M} e^{\frac{j2\pi\tau\|\mathbf{v}\|\cos(\theta_{n,m,AoA}-\theta_v)}{\lambda}} \qquad (3.76)
\end{aligned}
$$

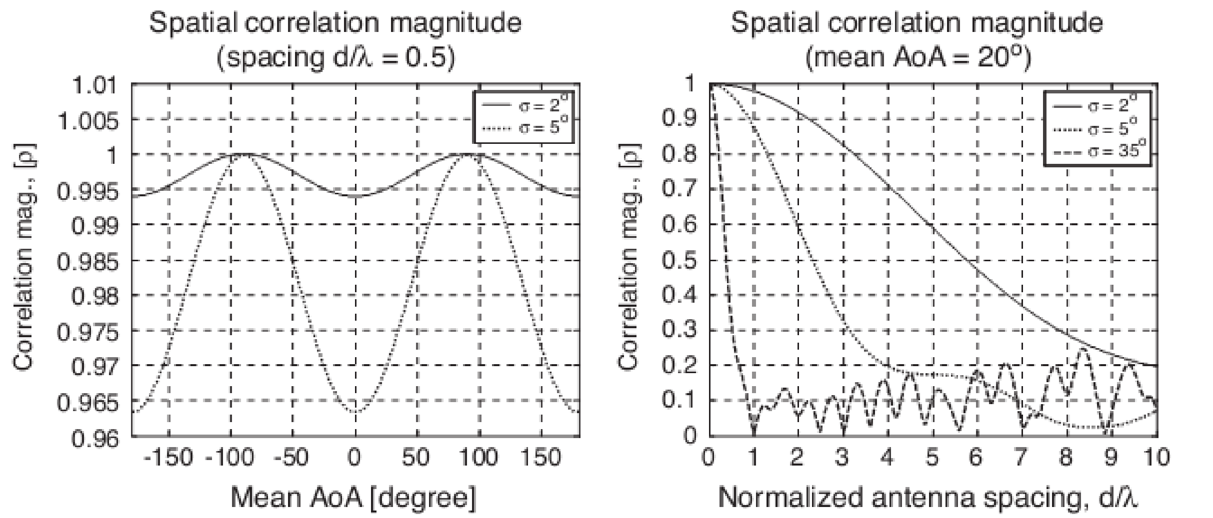
[redacted]

[redacted]

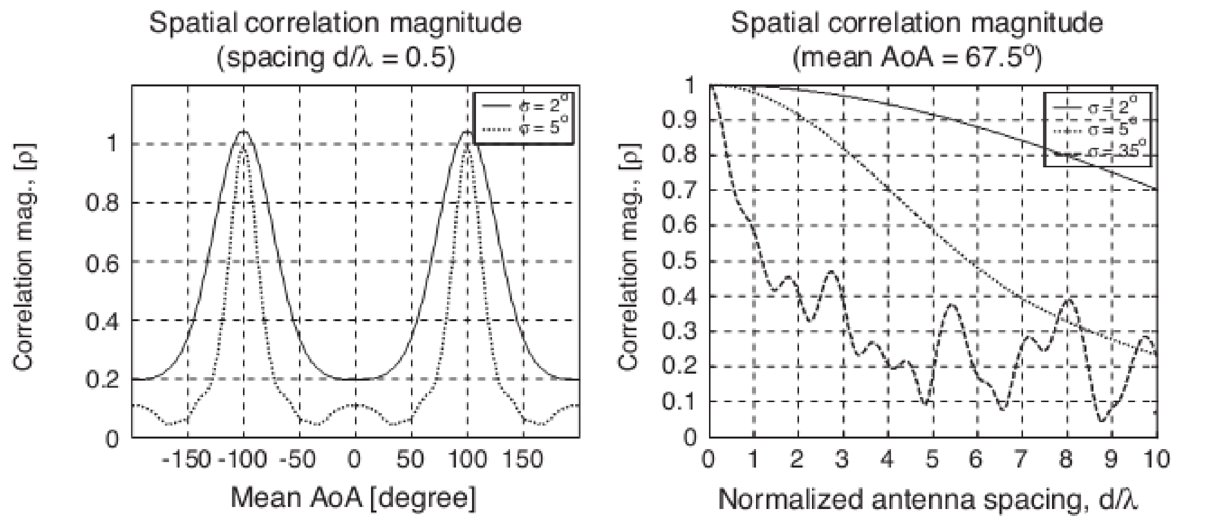
[redacted]

[redacted]

[redacted]

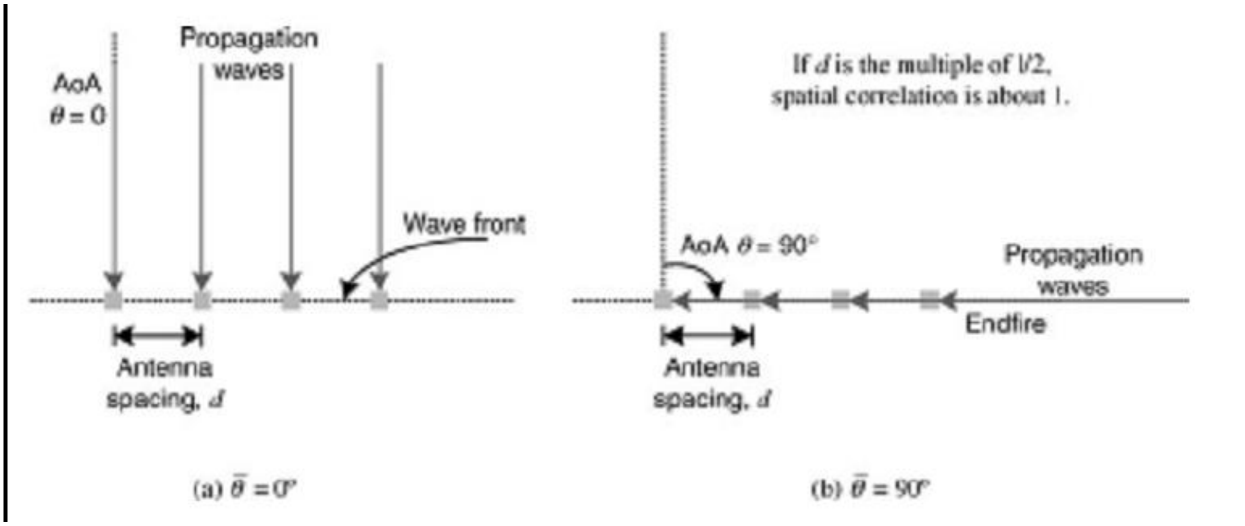


(a) Spatial correlation as AoA varies: d/λ=0.5 (b) Spatial correlation as antenna spacing varies: $\bar{\theta} = 20^{\circ}$

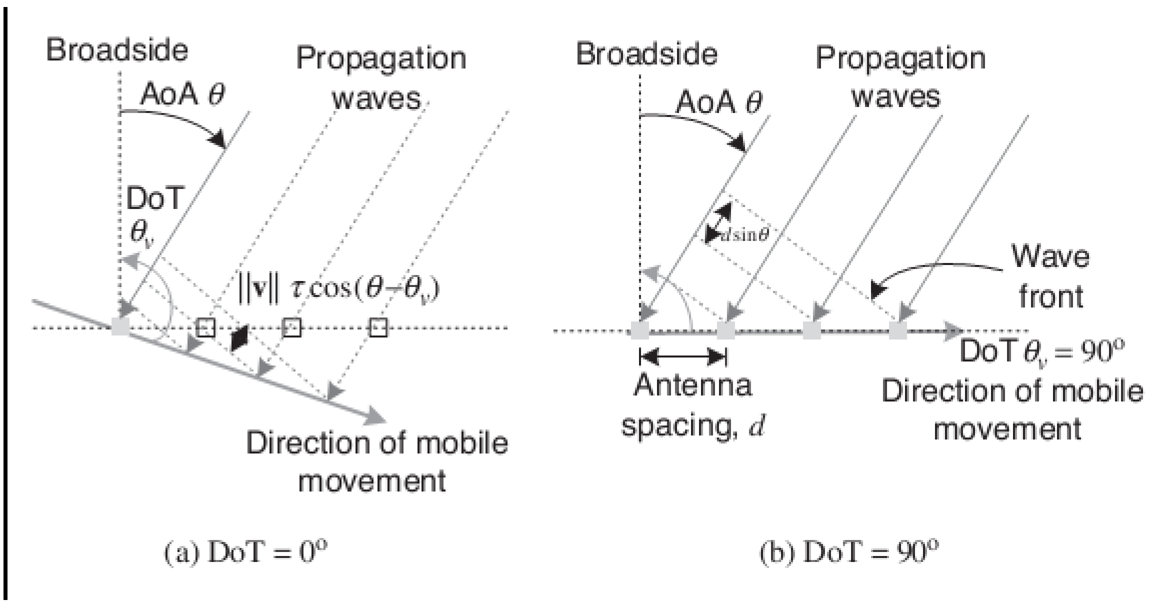


(c) Spatial correlation as AoA varies: d/λ=10 (d) Spatial correlation as antenna spacing varies: $\bar{\theta} = 20^{\circ}$

[redacted]

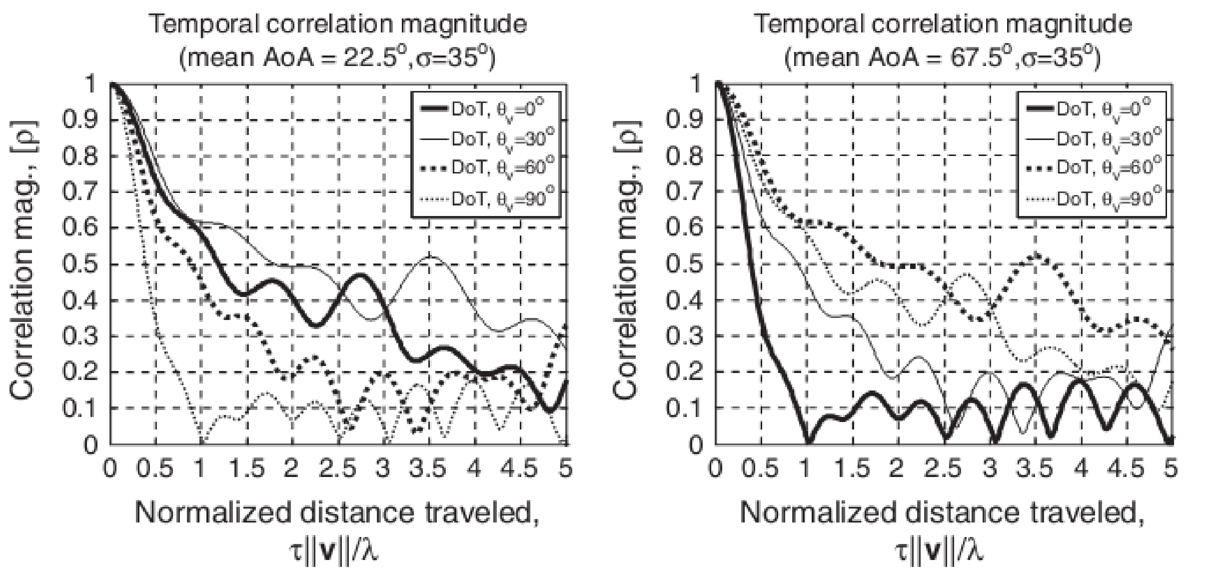


[redacted]



[redacted]

$$
\begin{aligned}
\rho^{tc}_{SCM}(\tau) &= \frac{1}{M}\sum_{m=1}^{M} e^{\frac{j2\pi\tau\|\mathbf{v}\|\cos(\theta_{n,m,AoA}-\theta_v)}{\lambda}} \\
&= \frac{1}{M}\sum_{m=1}^{M} e^{\frac{j2\pi\tau\|\mathbf{v}\|\sin\theta_{n,m,AoA}}{\lambda}} \\
&\overset{d=\tau\|\mathbf{v}\|}{=} \frac{1}{M}\sum_{m=1}^{M} e^{\frac{j2\pi d\ \sin\theta_{n,m,AoA}}{\lambda}} \\
&= \rho^{sc}_{SCM}(d)
\end{aligned}
\tag{3.77}
$$



(a) $\bar{\theta}$=22.5 (b) $\bar{\theta}$=67.5

[redacted]

[redacted]

[redacted]

[redacted]

[redacted]

[redacted]

[redacted]

[redacted]

[redacted]

[redacted]

[redacted]

[redacted]

[redacted]

$$\mathbf{H} = \mathbf{U}\mathbf{\Sigma}\mathbf{V}^H \tag{9.1}$$

[redacted]

$$\begin{aligned}\mathbf{H} &= \mathbf{U}\mathbf{\Sigma}\mathbf{V}^H \\ &= \underbrace{\left[\mathbf{U}_{N_{\min}}\ \mathbf{U}_{N_R-N_{\min}}\right]}_{\mathbf{U}} \underbrace{\begin{bmatrix}\mathbf{\Sigma}_{N_{\min}} \\ \mathbf{0}_{N_R-N_{\min}}\end{bmatrix}}_{\Sigma} \mathbf{V}^H \\ &= \mathbf{U}_{N_{\min}}\mathbf{\Sigma}_{N_{\min}}\mathbf{V}^H\end{aligned} \tag{9.2}$$

[redacted]

[redacted]

$$\mathbf{H} = \mathbf{U}\underbrace{[\mathbf{\Sigma}_{N_{\min}} \; \mathbf{0}_{N_T-N_{\min}}]}_{\mathbf{\Sigma}} \underbrace{\begin{bmatrix} \mathbf{V}^H_{N_{\min}} \\ \mathbf{V}^H_{N_T-N_{\min}} \end{bmatrix}}_{\mathbf{V}^H} = \mathbf{U}\mathbf{\Sigma}_{N_{\min}}\mathbf{V}^H_{N_{\min}} \tag{9.3}$$

[redacted]

$$\mathbf{H}\mathbf{H}^H = \mathbf{U}\mathbf{\Sigma}\mathbf{\Sigma}^H\mathbf{U}^H = \mathbf{Q}\mathbf{\Lambda}\mathbf{Q}^H \tag{9.4}$$

[redacted]

$$\lambda_i = \begin{cases} \sigma_i^2, & \text{if } i = 1, 2, \cdots, N_{\min} \\ 0, & \text{if } i = N_{\min}+1, \cdots, N_R. \end{cases} \tag{9.5}$$

[redacted]

$$\mathbf{H}\underbrace{[\mathbf{x}_1 \; \mathbf{x}_2 \cdots \mathbf{x}_n]}_{\mathbf{X}} = \underbrace{[\mathbf{x}_1 \; \mathbf{x}_2 \cdots \; \mathbf{x}_n]}_{\mathbf{X}} \mathbf{\Lambda}_{\text{non-}H} \tag{9.6}$$

[redacted]

$$\mathbf{H} = \mathbf{X}\mathbf{\Lambda}_{\text{non-}H}\mathbf{X}^{-1} \tag{9.7}$$

[redacted]

$$\|\mathbf{H}\|_F^2 = \mathrm{Tr}(\mathbf{H}\mathbf{H}^H) = \sum_{i=1}^{N_R} \sum_{j=1}^{N_T} \left|h_{i,j}\right|^2. \tag{9.8}$$

[redacted]

$$\begin{aligned} \|\mathbf{H}\|_F^2 &= \left\|\mathbf{Q}^H\mathbf{H}\right\|_F^2 \\ &= \mathrm{Tr}\left(\mathbf{Q}^H\mathbf{H}\mathbf{H}^H\mathbf{Q}\right) \\ &= \mathrm{Tr}\left(\mathbf{Q}^H\mathbf{Q}\mathbf{\Lambda}\mathbf{Q}^H\mathbf{Q}\right) \\ &= \mathrm{Tr}(\mathbf{\Lambda}) \\ &= \sum_{i=1}^{N_{\min}} \lambda_i \\ &= \sum_{i=1}^{N_{\min}} \sigma_i^2 \end{aligned} \tag{9.9}$$

[redacted]

$$\mathbf{y} = \sqrt{\frac{\mathsf{E}_x}{N_T}}\mathbf{H}\mathbf{x} + \mathbf{z} \tag{9.10}$$

[redacted]

$$\mathbf{R}_{xx} = E\{\mathbf{x}\mathbf{x}^H\}. \tag{9.11}$$

[redacted]

$$C = \max_{f(\mathbf{x})} I(\mathbf{x};\mathbf{y}) \text{ bits/channel use} \tag{9.12}$$

[redacted]

$$I(\mathbf{x};\mathbf{y}) = H(\mathbf{y}) - H(\mathbf{y}|\mathbf{x}) \tag{9.13}$$

[redacted]

$$H(\mathbf{y}|\mathbf{x}) = H(\mathbf{z}) \tag{9.14}$$

[redacted]

$$I(\mathbf{x};\mathbf{y}) = H(\mathbf{y}) - H(\mathbf{z}) \tag{9.15}$$

[redacted]

$$\begin{aligned}
\mathbf{R}_{yy} &= E\{\mathbf{y}\mathbf{y}^H\} = E\left\{\left(\sqrt{\frac{\mathsf{E}_x}{N_T}}\mathbf{H}\mathbf{x} + \mathbf{z}\right)\left(\sqrt{\frac{\mathsf{E}_x}{N_T}}\mathbf{x}^H\mathbf{H}^H + \mathbf{z}^H\right)\right\} \\
&= E\left\{\left(\frac{\mathsf{E}_x}{N_T}\mathbf{H}\mathbf{x}\mathbf{x}^H\mathbf{H}^H + \mathbf{z}\mathbf{z}^H\right)\right\} \\
&= \frac{\mathsf{E}_x}{N_T}E\{\mathbf{H}\mathbf{x}\mathbf{x}^H\mathbf{H}^H + \mathbf{z}\mathbf{z}^H\} \\
&= \frac{\mathsf{E}_x}{N_T}\mathbf{H}E\{\mathbf{x}\mathbf{x}^H\}\mathbf{H}^H + E\{\mathbf{z}\mathbf{z}^H\} \\
&= \frac{\mathsf{E}_x}{N_T}\mathbf{H}\mathbf{R}_{xx}\mathbf{H}^H + \mathsf{N}_0\mathbf{I}_{N_R}
\end{aligned} \tag{9.16}$$

[redacted]

$$\begin{aligned}
H(\mathbf{y}) &= \log_2\{\det(\pi e \mathbf{R}_{yy})\} \\
H(\mathbf{z}) &= \log_2\{\det(\pi e \mathsf{N}_0 \mathbf{I}_{N_R})\}
\end{aligned} \tag{9.17}$$

[redacted]

$$I(\mathbf{x};\mathbf{y}) = \log_2 \det\left(\mathbf{I}_{N_R} + \frac{\mathsf{E}_x}{N_T\mathsf{N}_0}\mathbf{H}\mathbf{R}_{xx}\mathbf{H}^H\right) \text{ bps/Hz.} \tag{9.18}$$

[redacted]

$$C = \max_{Tr(\mathbf{R}_{xx})=N_T} \log_2 \det\left(\mathbf{I}_{N_R} + \frac{\mathsf{E}_x}{N_T \mathsf{N}_0}\mathbf{H}\mathbf{R}_{xx}\mathbf{H}^H\right) \text{bps/Hz}. \quad (9.19)$$

[redacted]

$$\tilde{\mathbf{y}} = \sqrt{\frac{\mathsf{E}_x}{N_T}}\mathbf{U}^H\mathbf{H}\mathbf{V}\tilde{\mathbf{x}} + \tilde{\mathbf{z}} \quad (9.20)$$

[redacted]

$$\tilde{\mathbf{y}} = \sqrt{\frac{\mathsf{E}_x}{N_T}}\mathbf{\Sigma}\tilde{\mathbf{x}} + \tilde{\mathbf{z}}$$

[redacted]

$$\tilde{y}_i = \sqrt{\frac{\mathsf{E}_x}{N_T}}\sqrt{\lambda_i}\tilde{x}_i + \tilde{z}_i, \quad i = 1, 2, \cdots, r. \quad (9.21)$$

[redacted]

$$C_i(\gamma_i) = \log_2\left(1 + \frac{\mathsf{E}_x \gamma_i}{N_T \mathsf{N}_0}\lambda_i\right), \quad i = 1, 2, \cdots, r. \quad (9.22)$$

[redacted]

$$E\{\mathbf{x}^H\mathbf{x}\} = \sum_{i=1}^{N_T} E\left\{|x_i|^2\right\} = N_T. \tag{9.23}$$

[redacted]

[redacted]

$$C = \sum_{i=1}^{r} C_i(\gamma_i) = \sum_{i=1}^{r} \log_2\left(1 + \frac{\mathsf{E}_\mathrm{x}\gamma_i}{N_T\mathsf{N}_0}\lambda_i\right) \tag{9.24}$$

[redacted]

$$C = \max_{\{\gamma_i\}} \sum_{i=1}^{r} \log_2\left(1 + \frac{\mathsf{E}_\mathrm{x}\gamma_i}{N_T\mathsf{N}_0}\lambda_i\right) \tag{9.25}$$

[redacted]

[redacted]

$$\gamma_i^{opt} = \left(\mu - \frac{N_T\mathsf{N}_0}{\mathsf{E}_\mathrm{x}\lambda_i}\right)^+, \quad i = 1, \cdots, r \tag{9.26}$$

$$\sum_{i=1}^{r} \gamma_i^{opt} = N_T. \tag{9.27}$$

[redacted]

$$(x)^+ = \begin{cases} x & \text{if } x \geq 0 \\ 0 & \text{if } x < 0 \end{cases}. \tag{9.28}$$

[redacted]

[redacted]

[redacted]

[redacted]

[redacted]

$$\mathbf{R}_{xx} = \mathbf{I}_{N_T} \tag{9.29}$$

[redacted]

$$C = \log_2 \det\left(\mathbf{I}_{N_R} + \frac{\mathsf{E}_x}{N_T \mathsf{N}_0}\mathbf{H}\mathbf{H}^H\right). \tag{9.30}$$

[redacted]

$$C = \log_2 \det\left(\mathbf{I}_{N_R} + \frac{\mathsf{E}_x}{N_T \mathsf{N}_0}\mathbf{Q}\boldsymbol{\Lambda}\mathbf{Q}^H\right) = \log_2 \det\left(\mathbf{I}_{N_R} + \frac{\mathsf{E}_x}{N_T \mathsf{N}_0}\boldsymbol{\Lambda}\right)$$
$$= \sum_{i=1}^{r} \log_2\left(1 + \frac{\mathsf{E}_x}{N_T \mathsf{N}_0}\lambda_i\right) \tag{9.31}$$

[redacted]

[redacted]

$$\lambda_i = \frac{\zeta}{N}, \quad i = 1, 2, \cdots, N. \tag{9.32}$$

[redacted]

$$\mathbf{H}\mathbf{H}^H = \mathbf{H}^H\mathbf{H} = \frac{\zeta}{N}\mathbf{I}_N \tag{9.33}$$

[redacted]

$$C = N \log_2\left(1 + \frac{\zeta \mathsf{E}_x}{\mathsf{N}_0 N}\right). \tag{9.34}$$

[redacted]

[redacted]

$$C_{SIMO} = \log_2\left(1 + \frac{\mathsf{E}_x}{\mathsf{N}_0}\|\mathbf{h}\|_F^2\right). \quad (9.35)$$

[redacted]

$$C_{SIMO} = \log_2\left(1 + \frac{\mathsf{E}_x}{\mathsf{N}_0}N_R\right). \quad (9.36)$$

[redacted]

$$C_{MISO} = \log_2\left(1 + \frac{\mathsf{E}_x}{N_T\mathsf{N}_0}\|\mathbf{h}\|_F^2\right). \quad (9.37)$$

[redacted]

$$C_{MISO} = \log_2\left(1 + \frac{\mathsf{E}_x}{\mathsf{N}_0}\right). \quad (9.38)$$

[redacted]

[redacted]

[redacted]

$$y = \sqrt{\mathsf{E}_{\mathrm{x}}}\mathbf{h} \cdot \frac{\mathbf{h}^H}{\|\mathbf{h}\|} x + z = \sqrt{\mathsf{E}_{\mathrm{x}}}\|\mathbf{h}\| x + z \tag{9.39}$$

[redacted]

$$C_{MISO} = \log_2\left(1 + \frac{\mathsf{E}_{\mathrm{x}}}{\mathsf{N}_0}\|\mathbf{h}\|_F^2\right) = \log_2\left(1 + \frac{\mathsf{E}_{\mathrm{x}}}{\mathsf{N}_0} N_T\right). \tag{9.40}$$

[redacted]

[redacted]

$$\overline{C} = E\{C(\mathbf{H})\} = E\left\{\max_{\mathrm{Tr}(\mathbf{R}_{xx}) = N_T} \log_2 \det\left(\mathbf{I}_{N_R} + \frac{\mathsf{E}_{\mathrm{x}}}{N_T \mathsf{N}_0}\mathbf{H}\mathbf{R}_{xx}\mathbf{H}^H\right)\right\} \tag{9.41}$$

[redacted]

$$\overline{C_{OL}} = E\left\{\sum_{i=1}^{r} \log_2\left(1 + \frac{\mathsf{E}_{\mathrm{x}}}{N_T \mathsf{N}_0}\lambda_i\right)\right\}. \tag{9.42}$$

[redacted]

$$\overline{C}_{CL} = E\left\{\max_{\sum_{i=1}^{r}\gamma_i=N_T}\sum_{i=1}^{r}\log_2\left(1+\frac{\mathsf{E}_x}{N_T\mathsf{N}_0}\gamma_i\lambda_i\right)\right\} \tag{9.43}$$

$$= E\left\{\sum_{i=1}^{r}\log_2\left(1+\frac{\mathsf{E}_x}{N_T\mathsf{N}_0}\gamma_i^{opt}\lambda_i\right)\right\}. \tag{9.44}$$

[redacted]

$$P_{out}(R) = \Pr(C(\mathbf{H})<R) \tag{9.45}$$

[redacted]

[redacted]

[redacted]

[redacted]

[redacted]

[redacted]

[redacted]

[redacted]

[redacted]

[redacted]

[redacted]

[redacted]

[redacted]

[redacted]

[redacted]

[redacted]

$$C \approx \max_{\mathrm{Tr}(\mathbf{R}_{xx})=N} \log_2 \det(\mathbf{R}_{xx}) + \log_2 \det\left(\frac{\mathsf{E}_x}{N\mathsf{N}_0}\mathbf{H}_w\mathbf{H}_w^H\right) \tag{9.46}$$

[redacted]

$$\mathbf{H} = \mathbf{R}_r^{1/2}\mathbf{H}_w\mathbf{R}_t^{1/2} \tag{9.47}$$

[redacted]

$$C = \log_2 \det\left(\mathbf{I}_{N_R} + \frac{\mathsf{E}_x}{N_T\mathsf{N}_0}\mathbf{R}_r^{1/2}\mathbf{H}_w\mathbf{R}_t\mathbf{H}_w^H\mathbf{R}_r^{H/2}\right). \tag{9.48}$$

[redacted]

$$C \approx \log_2 \det\left(\frac{\mathsf{E}_x}{N_T\mathsf{N}_0}\mathbf{H}_w\mathbf{H}_w^H\right) + \log_2 \det(\mathbf{R}_r) + \log_2 \det(\mathbf{R}_t). \tag{9.49}$$

[redacted]

$$\log_2 \det(\mathbf{R}_r) + \log_2 \det(\mathbf{R}_t). \tag{9.50}$$

[redacted]

$$\det(\mathbf{R}) = \prod_{i=1}^{N} \lambda_i. \tag{9.51}$$

[redacted]

$$\left(\Pi_{i=1}^{N} \lambda_i\right)^{\frac{1}{N}} \leq \frac{1}{N} \sum_{i=1}^{N} \lambda_i = 1. \tag{9.52}$$

[redacted]

$$\log_2 \det(\mathbf{R}) \leq 0 \tag{9.53}$$

[redacted]

[redacted]

$$\mathbf{R}_t = \begin{bmatrix} 1 & 0.76e^{j0.17\pi} & 0.43e^{j0.35\pi} & 0.25e^{j0.53\pi} \\ 0.76e^{-j0.17\pi} & 1 & 0.76e^{j0.17\pi} & 0.43e^{j0.35\pi} \\ 0.43e^{-j0.35\pi} & 0.76e^{-j0.17\pi} & 1 & 0.76e^{j0.17\pi} \\ 0.25e^{-j0.53\pi} & 0.43e^{-j0.35\pi} & 0.76e^{-j0.17\pi} & 1 \end{bmatrix} \tag{9.54}$$

[redacted]

[redacted]

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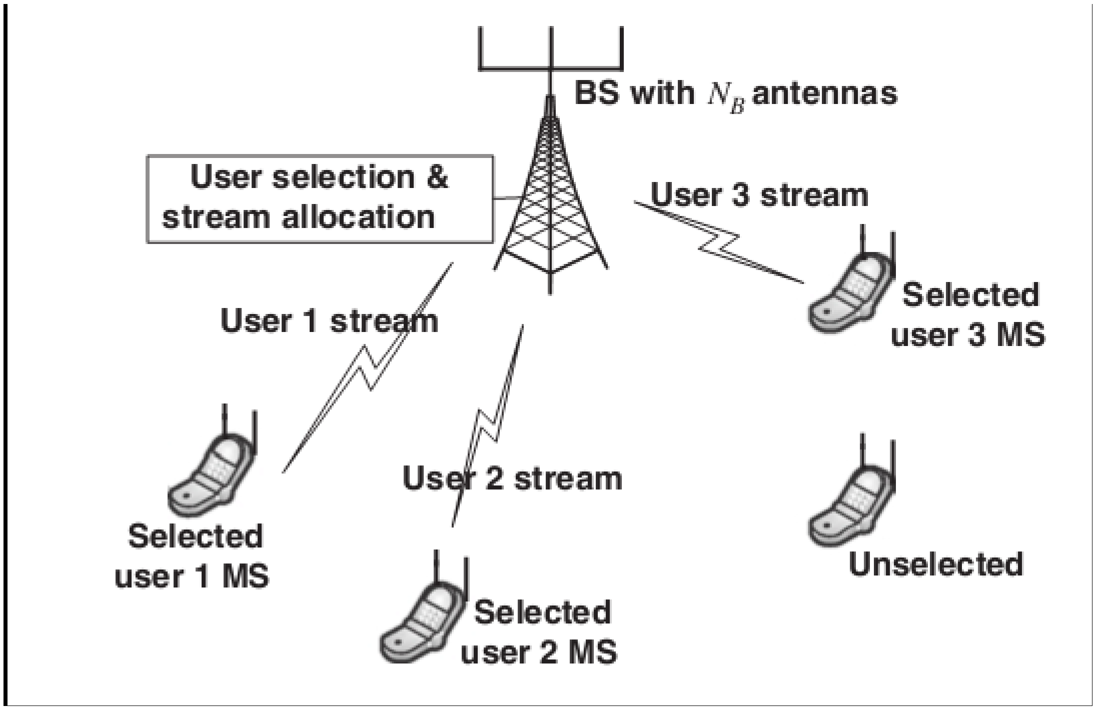
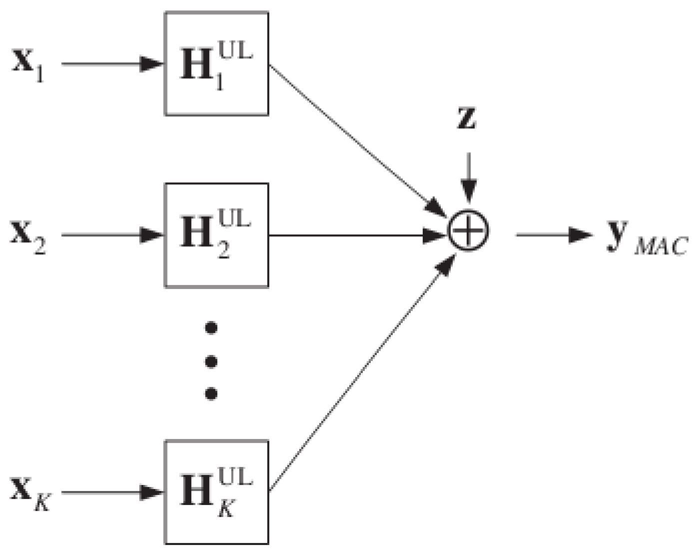


[redacted]

[redacted]

[redacted]

$$
\begin{aligned}
\mathbf{y}_{MAC} &= \mathbf{H}_1^{\mathrm{UL}}\mathbf{x}_1 + \mathbf{H}_2^{\mathrm{UL}}\mathbf{x}_2 + \cdots + \mathbf{H}_K^{\mathrm{UL}}\mathbf{x}_K + \mathbf{z} \\
&= \underbrace{\left[\mathbf{H}_1^{\mathrm{UL}}\ \mathbf{H}_2^{\mathrm{UL}} \cdots \mathbf{H}_K^{\mathrm{UL}}\right]}_{=\mathbf{H}^{\mathrm{UL}}} \begin{bmatrix} \mathbf{x}_1 \\ \vdots \\ \mathbf{x}_K \end{bmatrix} + \mathbf{z} = \mathbf{H}^{\mathrm{UL}} \begin{bmatrix} \mathbf{x}_1 \\ \vdots \\ \mathbf{x}_K \end{bmatrix} + \mathbf{z}
\end{aligned} \tag{13.1}
$$



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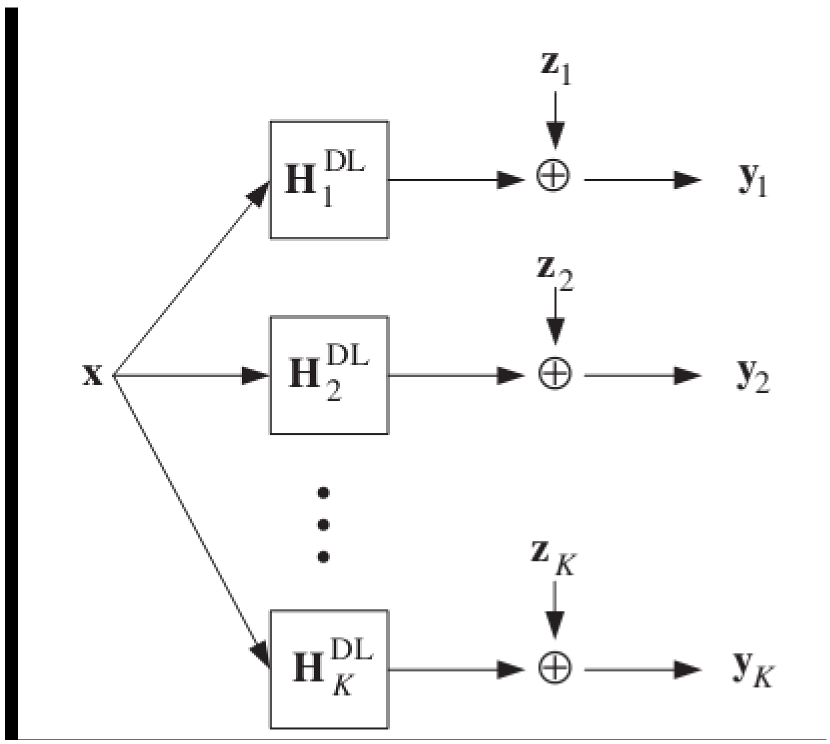
[redacted]

[redacted]

$$
\mathbf{y}_u = \mathbf{H}_u^{\mathrm{DL}}\mathbf{x} + \mathbf{z}_u, \quad u = 1, 2, \cdots, K \tag{13.2}
$$

[redacted]

$$\underbrace{\begin{bmatrix} \mathbf{y}_1 \\ \mathbf{y}_2 \\ \vdots \\ \mathbf{y}_K \end{bmatrix}}_{\mathbf{y}_{BC}} = \underbrace{\begin{bmatrix} \mathbf{H}_1^{\mathrm{DL}} \\ \mathbf{H}_2^{\mathrm{DL}} \\ \vdots \\ \mathbf{H}_K^{\mathrm{DL}} \end{bmatrix}}_{\mathbf{H}_{\mathrm{DL}}} \mathbf{x} + \underbrace{\begin{bmatrix} \mathbf{z}_1 \\ \mathbf{z}_2 \\ \vdots \\ \mathbf{z}_K \end{bmatrix}}_{\mathbf{z}} \tag{13.3}$$



[redacted]

[redacted]

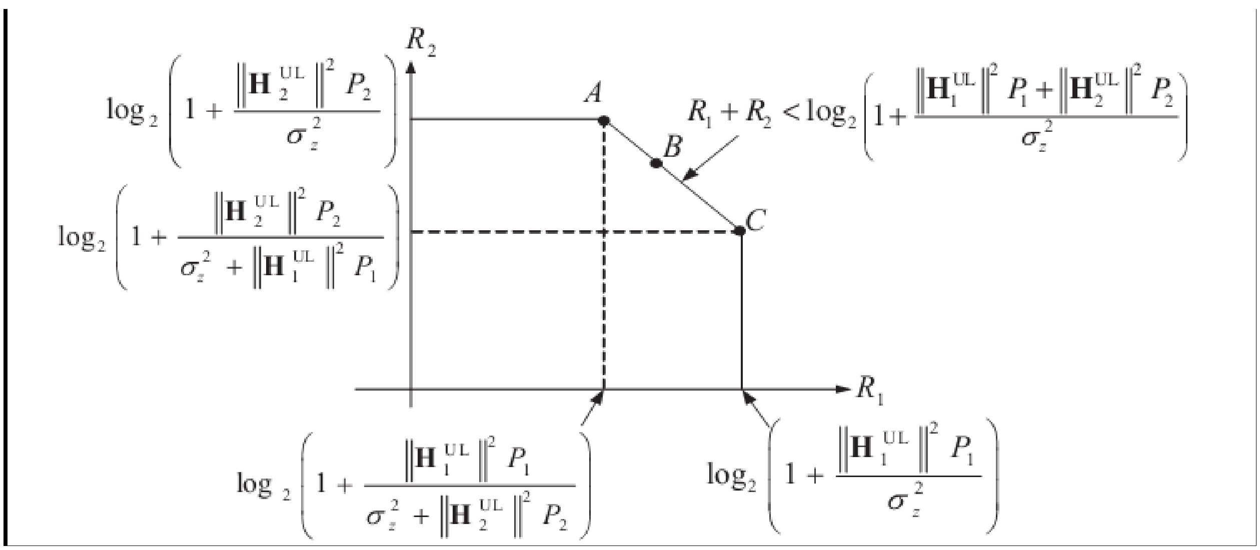
[redacted]

[redacted]

[redacted]

$$
\begin{cases}
R_1 \leq \log_2\left(1 + \left\|\mathbf{H}_1^{\mathrm{UL}}\right\|^2 P_1\right) \\
R_2 \leq \log_2\left(1 + \left\|\mathbf{H}_2^{\mathrm{UL}}\right\|^2 P_2\right) \\
R_1 + R_2 \leq \log_2\left(1 + \left\|\mathbf{H}_1^{\mathrm{UL}}\right\|^2 P_1 + \left\|\mathbf{H}_2^{\mathrm{UL}}\right\|^2 P_2\right)
\end{cases}
\tag{13.4}
$$

[redacted]



[redacted]

[redacted]

$$\begin{aligned}\mathbf{y}_{MAC} &= \mathbf{H}_1^{\mathrm{UL}} x_1 + \mathbf{H}_2^{\mathrm{UL}} x_2 + \begin{bmatrix} z_1 \\ z_2 \end{bmatrix} \\ &= \left[\mathbf{H}_1^{\mathrm{UL}}\ \mathbf{H}_2^{\mathrm{UL}}\right] \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} z_1 \\ z_2 \end{bmatrix}\end{aligned} \tag{13.5}$$

[redacted]

$$\tilde{\mathbf{y}}_{MAC} = \mathbf{y}_{MAC} - \mathbf{H}_1^{\mathrm{UL}} x_1 \ = \mathbf{H}_2^{\mathrm{UL}} x_2 + \begin{bmatrix} z_1 \\ z_2 \end{bmatrix} \tag{13.6}$$

[redacted]

[redacted]

[redacted]

$$\underbrace{\begin{bmatrix} y_1 \\ y_2 \end{bmatrix}}_{\mathbf{y}_{\mathrm{BC}}} = \underbrace{\begin{bmatrix} \mathbf{H}_1^{\mathrm{DL}} \\ \mathbf{H}_2^{\mathrm{DL}} \end{bmatrix}}_{\mathbf{H}^{\mathrm{DL}}} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} z_1 \\ z_2 \end{bmatrix} \tag{13.7}$$

[redacted]

$$\mathbf{H}^{\mathrm{DL}} = \underbrace{\begin{bmatrix} l_{11} & 0 \\ l_{21} & l_{22} \end{bmatrix}}_{\mathbf{L}} \underbrace{\begin{bmatrix} \mathbf{q}_1 \\ \mathbf{q}_2 \end{bmatrix}}_{\mathbf{Q}} \tag{13.8}$$

[redacted]

$$\begin{aligned} l_{11} &= \left\|\mathbf{H}_1^{\mathrm{DL}}\right\|, \mathbf{q}_1 = \frac{1}{l_{11}} \mathbf{H}_1^{\mathrm{DL}}, \\ l_{21} &= \mathbf{q}_1 \cdot \left(\mathbf{H}_2^{\mathrm{DL}}\right)^H, \\ l_{22} &= \left\|\mathbf{H}_2^{\mathrm{DL}} - l_{21}\mathbf{q}_1\right\|, \\ \text{and} \quad \mathbf{q}_2 &= \frac{1}{l_{22}} \left(\mathbf{H}_2^{\mathrm{DL}} - l_{21}\mathbf{q}_1\right). \end{aligned}$$

[redacted]

$$\underbrace{\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}}_{\mathbf{x}} = \mathbf{Q}^H \begin{bmatrix} \tilde{x}_1 \\ \tilde{x}_2 - \frac{1}{l_{22}} l_{21} \tilde{x}_1 \end{bmatrix} \tag{13.9}$$

[redacted]

$$
\begin{aligned}
\mathbf{y}_{\mathrm{BC}} &= \mathbf{H}^{\mathrm{DL}}\mathbf{x}+\mathbf{z} \\
&= \begin{bmatrix} l_{11} & 0 \\ l_{21} & l_{22} \end{bmatrix} \begin{bmatrix} \mathbf{q}_1 \\ \mathbf{q}_2 \end{bmatrix} \begin{bmatrix} \mathbf{q}_1^H \, \mathbf{q}_2^H \end{bmatrix} \begin{bmatrix} \tilde{x}_1 \\ \tilde{x}_2 - \dfrac{1}{l_{22}} l_{21} \tilde{x}_1 \end{bmatrix} + \mathbf{z} \\
&= \begin{bmatrix} l_{11} & 0 \\ 0 & l_{22} \end{bmatrix} \begin{bmatrix} \tilde{x}_1 \\ \tilde{x}_2 \end{bmatrix} + \mathbf{z} \\
&= \begin{bmatrix} \left\|\mathbf{H}_1^{\mathrm{DL}}\right\| & 0 \\ 0 & \left\|\mathbf{H}_2^{\mathrm{DL}} - l_{12}\mathbf{q}_1\right\| \end{bmatrix} \begin{bmatrix} \tilde{x}_1 \\ \tilde{x}_2 \end{bmatrix} + \mathbf{z}
\end{aligned}
\tag{13.10}
$$

[redacted]

$$
E\left\{|x_1|^2\right\} = E\left\{|\tilde{x}_1|^2\right\} = \alpha P
$$

$$
\text{and} \quad E\left\{|x_2|^2\right\} = E\left\{\left|\tilde{x}_2 - \frac{l_{21}}{l_{22}}\tilde{x}_1\right|^2\right\} = (1-\alpha)P, \quad \alpha \in [0,1].
$$

[redacted]

$$
R_1 = \log\left(1 + \left\|\mathbf{H}_1^{\mathrm{DL}}\right\|^2 \frac{\alpha P}{\sigma_z^2}\right), \tag{13.11}
$$

[redacted]

$$
R_2 = \log_2\left(1 + \left\|\mathbf{H}_2^{\mathrm{DL}} - l_{21}\mathbf{q}_1\right\|^2 \frac{(1-\alpha)P}{\sigma_z^2}\right). \tag{13.12}
$$

[redacted]

$$R_2 = \log_2\left(1 + \left\|\mathbf{H}_2^{\mathrm{DL}}\right\|^2 \frac{(1-\alpha)P}{\sigma_z^2}\right). \tag{13.13}$$

[redacted]

[redacted]

[redacted]

[redacted]

[redacted]

[redacted]

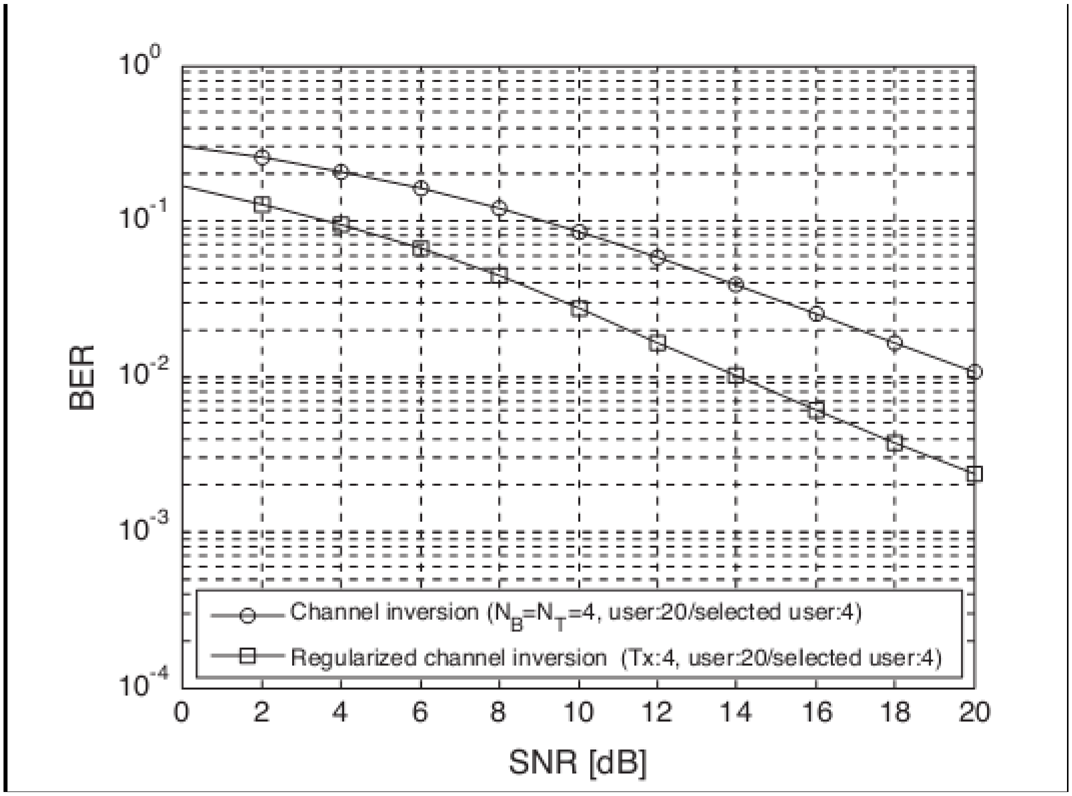
$$y_u = \mathbf{H}_u^{\mathrm{DL}} \begin{bmatrix} \tilde{x}_1 \\ \tilde{x}_2 \\ \vdots \\ \tilde{x}_K \end{bmatrix} + z_u, \quad u = 1, 2, \cdots, K. \tag{13.14}$$

[redacted]

$$
\underbrace{\begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_K \end{bmatrix}}_{\mathbf{y}_{\mathrm{BC}}} = \underbrace{\begin{bmatrix} \mathbf{H}_1^{\mathrm{DL}} \\ \mathbf{H}_2^{\mathrm{DL}} \\ \vdots \\ \mathbf{H}_K^{\mathrm{DL}} \end{bmatrix}}_{\mathbf{H}^{\mathrm{DL}}} \underbrace{\begin{bmatrix} \tilde{x}_1 \\ \tilde{x}_2 \\ \vdots \\ \tilde{x}_K \end{bmatrix}}_{\mathbf{x}} + \underbrace{\begin{bmatrix} z_1 \\ z_2 \\ \vdots \\ z_K \end{bmatrix}}_{\mathbf{z}} \tag{13.15}
$$

[redacted]

[redacted]



[redacted]

[redacted]

[redacted]

[redacted]

[redacted]

[redacted]

[redacted]

$$\begin{aligned}\mathbf{y}_u &= \mathbf{H}_u^{\mathrm{DL}} \sum_{k=1}^{K} \mathbf{W}_k \tilde{\mathbf{x}}_k + \mathbf{z}_u \\ &= \mathbf{H}_u^{\mathrm{DL}} \mathbf{W}_u \tilde{\mathbf{x}}_u + \sum_{k=1.\ k \neq u}^{K} \mathbf{H}_u^{\mathrm{DL}} \mathbf{W}_k \tilde{\mathbf{x}}_k + \mathbf{z}_u\end{aligned} \tag{13.16}$$

[redacted]

$$\begin{aligned}\begin{bmatrix}\mathbf{y}_1\\ \mathbf{y}_2\\ \mathbf{y}_3\end{bmatrix} &= \underbrace{\begin{bmatrix}\mathbf{H}_1^{\mathrm{DL}} & \mathbf{H}_1^{\mathrm{DL}} & \mathbf{H}_1^{\mathrm{DL}}\\ \mathbf{H}_2^{\mathrm{DL}} & \mathbf{H}_2^{\mathrm{DL}} & \mathbf{H}_2^{\mathrm{DL}}\\ \mathbf{H}_3^{\mathrm{DL}} & \mathbf{H}_3^{\mathrm{DL}} & \mathbf{H}_3^{\mathrm{DL}}\end{bmatrix}}_{\mathbf{H}_{\mathrm{DL}}} \underbrace{\begin{bmatrix}\mathbf{W}_1\tilde{\mathbf{x}}_1\\ \mathbf{W}_2\tilde{\mathbf{x}}_2\\ \mathbf{W}_3\tilde{\mathbf{x}}_3\end{bmatrix}}_{\mathbf{x}} + \begin{bmatrix}\mathbf{z}_1\\ \mathbf{z}_2\\ \mathbf{z}_3\end{bmatrix} \\ &= \begin{bmatrix}\mathbf{H}_1^{\mathrm{DL}}\mathbf{W}_1 & \mathbf{H}_1^{\mathrm{DL}}\mathbf{W}_2 & \mathbf{H}_1^{\mathrm{DL}}\mathbf{W}_3\\ \mathbf{H}_2^{\mathrm{DL}}\mathbf{W}_1 & \mathbf{H}_2^{\mathrm{DL}}\mathbf{W}_2 & \mathbf{H}_2^{\mathrm{DL}}\mathbf{W}_3\\ \mathbf{H}_3^{\mathrm{DL}}\mathbf{W}_1 & \mathbf{H}_3^{\mathrm{DL}}\mathbf{W}_2 & \mathbf{H}_3^{\mathrm{DL}}\mathbf{W}_3\end{bmatrix} \begin{bmatrix}\tilde{\mathbf{x}}_1\\ \tilde{\mathbf{x}}_2\\ \tilde{\mathbf{x}}_3\end{bmatrix} + \begin{bmatrix}\mathbf{z}_1\\ \mathbf{z}_2\\ \mathbf{z}_3\end{bmatrix}\end{aligned} \tag{13.17}$$

[redacted]

$$\mathbf{H}_u^{\mathrm{DL}}\mathbf{W}_k = \mathbf{0}_{N_{M,u}\times N_{M,u}}, \forall\, u \neq k \tag{13.18}$$

[redacted]

$$\mathbf{y}_u = \mathbf{H}_u^{\mathrm{DL}}\mathbf{W}_u\tilde{\mathbf{x}}_u + \mathbf{z}_u, \quad u = 1, 2, \cdots, K \tag{13.19}$$

[redacted]

[redacted]

$$\tilde{\mathbf{H}}_u^{\mathrm{DL}} = \left[\left(\mathbf{H}_1^{\mathrm{DL}}\right)^H \cdots \left(\mathbf{H}_{u-1}^{\mathrm{DL}}\right)^H \left(\mathbf{H}_{u+1}^{\mathrm{DL}}\right)^H \cdots \left(\mathbf{H}_K^{\mathrm{DL}}\right)^H\right]^H \tag{13.20}$$

[redacted]

$$\tilde{\mathbf{H}}_u^{\mathrm{DL}}\mathbf{W}_u = \mathbf{0}_{(N_{M,total}-N_{M,u})\times N_{M,u}}, \quad u = 1, 2, \cdots, K \tag{13.21}$$

[redacted]

$$\begin{bmatrix}\mathbf{y}_1\\ \mathbf{y}_2\\ \mathbf{y}_3\end{bmatrix} = \begin{bmatrix}\mathbf{H}_1^{\mathrm{DL}}\mathbf{W}_1 & \mathbf{0} & \mathbf{0}\\ \mathbf{0} & \mathbf{H}_2^{\mathrm{DL}}\mathbf{W}_2 & \mathbf{0}\\ \mathbf{0} & \mathbf{0} & \mathbf{H}_3^{\mathrm{DL}}\mathbf{W}_3\end{bmatrix}\begin{bmatrix}\tilde{\mathbf{x}}_1\\ \tilde{\mathbf{x}}_2\\ \tilde{\mathbf{x}}_3\end{bmatrix} + \begin{bmatrix}\mathbf{z}_1\\ \mathbf{z}_2\\ \mathbf{z}_3\end{bmatrix} \tag{13.22}$$

[redacted]

[redacted]

$$\tilde{\mathbf{H}}_u^{\mathrm{DL}} = \tilde{\mathbf{U}}_u \tilde{\Lambda}_u \left[\tilde{\mathbf{V}}_u^{\text{non-zero}} \; \tilde{\mathbf{V}}_u^{\text{zero}} \right]^H \tag{13.23}$$

[redacted]

$$\begin{aligned} \tilde{\mathbf{H}}_u^{\mathrm{DL}} \tilde{\mathbf{V}}_u^{\text{zero}} &= \tilde{\mathbf{U}}_u \left[\tilde{\Lambda}_u^{\text{non-zero}} \; \mathbf{0} \right] \begin{bmatrix} \left(\tilde{\mathbf{V}}_u^{\text{non-zero}} \right)^H \\ \left(\tilde{\mathbf{V}}_u^{\text{zero}} \right)^H \end{bmatrix} \tilde{\mathbf{V}}_u^{\text{zero}} \\ &= \tilde{\mathbf{U}}_u \tilde{\Lambda}_u^{\text{non-zero}} \left(\tilde{\mathbf{V}}_u^{\text{non-zero}} \right)^H \tilde{\mathbf{V}}_u^{\text{zero}} \\ &= \tilde{\mathbf{U}}_u \tilde{\Lambda}_u^{\text{non-zero}} \mathbf{0} \\ &= \mathbf{0} \end{aligned} \tag{13.24}$$

[redacted]

[redacted]

$$\begin{aligned}\tilde{\mathbf{H}}_1^{\mathrm{DL}} &= \tilde{\mathbf{U}}_1\tilde{\Lambda}_1\left[\tilde{\mathbf{V}}_1^{\text{non-zero}}\ \tilde{\mathbf{V}}_1^{\text{zero}}\right]^H \\ &= \left[\tilde{\mathbf{u}}_{11}\quad \tilde{\mathbf{u}}_{12}\right]\begin{bmatrix}\tilde{\lambda}_{11} & 0 & 0 & 0\\ 0 & \tilde{\lambda}_{12} & 0 & 0\end{bmatrix}\left[\tilde{\mathbf{v}}_{11}\quad \tilde{\mathbf{v}}_{12}\quad \tilde{\mathbf{v}}_{13}\quad \tilde{\mathbf{v}}_{14}\right]^H\end{aligned} \tag{13.25}$$

$$\begin{aligned}\tilde{\mathbf{H}}_2^{\mathrm{DL}} &= \tilde{\mathbf{U}}_2\tilde{\Lambda}_2\left[\tilde{\mathbf{V}}_2^{\text{non-zero}}\ \tilde{\mathbf{V}}_2^{\text{zero}}\right]^H \\ &= \left[\tilde{\mathbf{u}}_{21}\quad \tilde{\mathbf{u}}_{22}\right]\begin{bmatrix}\tilde{\lambda}_{21} & 0 & 0 & 0\\ 0 & \tilde{\lambda}_{22} & 0 & 0\end{bmatrix}\left[\tilde{\mathbf{v}}_{21}\quad \tilde{\mathbf{v}}_{22}\quad \tilde{\mathbf{v}}_{23}\quad \tilde{\mathbf{v}}_{24}\right]^H\end{aligned} \tag{13.26}$$

[redacted]

$$\begin{aligned}\mathbf{W}_1 &= \tilde{\mathbf{V}}_1^{\text{zero}} = \left[\tilde{\mathbf{v}}_{13}\quad \tilde{\mathbf{v}}_{14}\right] \\ \mathbf{W}_2 &= \tilde{\mathbf{V}}_2^{\text{zero}} = \left[\tilde{\mathbf{v}}_{23}\quad \tilde{\mathbf{v}}_{24}\right]\end{aligned} \tag{13.27}$$

[redacted]

$$\mathbf{x} = \mathbf{W}_1\tilde{\mathbf{x}}_1 + \mathbf{W}_2\tilde{\mathbf{x}}_2 \tag{13.28}$$

[redacted]

$$\begin{aligned}
\mathbf{y}_1 &= \mathbf{H}_1^{\mathrm{DL}}\mathbf{x} + \mathbf{z}_1 \\
&= \mathbf{H}_1^{\mathrm{DL}}(\mathbf{W}_1\tilde{\mathbf{x}}_1 + \mathbf{W}_2\tilde{\mathbf{x}}_2) + \mathbf{z}_1 \\
&= \tilde{\mathbf{H}}_2^{\mathrm{DL}}\left(\tilde{\mathbf{V}}_1^{\mathrm{zero}}\tilde{\mathbf{x}}_1 + \tilde{\mathbf{V}}_2^{\mathrm{zero}}\tilde{\mathbf{x}}_2\right) + \mathbf{z}_1 \qquad (13.29) \\
&= \tilde{\mathbf{H}}_2^{\mathrm{DL}}\tilde{\mathbf{V}}_1^{\mathrm{zero}}\tilde{\mathbf{x}}_1 + \mathbf{z}_1 \\
&= \mathbf{H}_1^{\mathrm{DL}}\tilde{\mathbf{V}}_1^{\mathrm{zero}}\tilde{\mathbf{x}}_1 + \mathbf{z}_1
\end{aligned}$$

[redacted]

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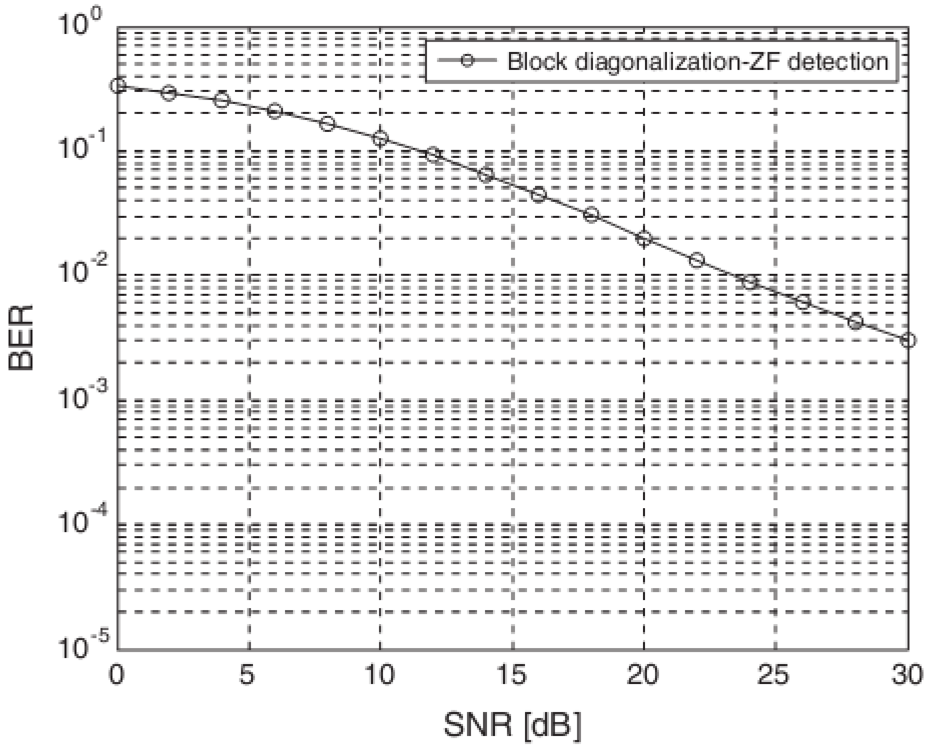


[redacted]

[redacted]

[redacted]

[redacted]

$$\begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \underbrace{\begin{bmatrix} \mathbf{H}_1^{\mathrm{DL}} \\ \mathbf{H}_2^{\mathrm{DL}} \\ \mathbf{H}_3^{\mathrm{DL}} \end{bmatrix}}_{\mathbf{H}^{\mathrm{DL}}} \begin{bmatrix} \tilde{x}_1 \\ \tilde{x}_2 \\ \tilde{x}_3 \end{bmatrix} + \begin{bmatrix} z_1 \\ z_2 \\ z_3 \end{bmatrix} \tag{13.30}$$

[redacted]

$$\mathbf{H}^{\mathrm{DL}} = \underbrace{\begin{bmatrix} l_{11} & 0 & 0 \\ l_{21} & l_{22} & 0 \\ l_{31} & l_{32} & l_{33} \end{bmatrix}}_{\mathbf{L}} \underbrace{\begin{bmatrix} \mathbf{q}_1 \\ \mathbf{q}_2 \\ \mathbf{q}_3 \end{bmatrix}}_{\mathbf{Q}} \tag{13.31}$$

[redacted]

$$\begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \underbrace{\begin{bmatrix} \mathbf{H}_1^{\mathrm{DL}} \\ \mathbf{H}_2^{\mathrm{DL}} \\ \mathbf{H}_3^{\mathrm{DL}} \end{bmatrix}}_{\mathbf{H}^{\mathrm{DL}}} \mathbf{Q}^H \mathbf{x} + \begin{bmatrix} z_1 \\ z_2 \\ z_3 \end{bmatrix} = \begin{bmatrix} l_{11} & 0 & 0 \\ l_{21} & l_{22} & 0 \\ l_{31} & l_{32} & l_{33} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} z_1 \\ z_2 \\ z_3 \end{bmatrix}. \tag{13.32}$$

[redacted]

$$y_1 = l_{11} x_1 + z_1. \tag{13.33}$$

[redacted]

$$x_1 = \tilde{x}_1 \tag{13.34}$$

[redacted]

$$y_2 = l_{21}x_1 + l_{22}x_2 + z_2 = l_{21}\tilde{x}_1 + l_{22}x_2 + z_2. \tag{13.35}$$

[redacted]

$$x_2 = \tilde{x}_2 - \frac{l_{21}}{l_{22}}x_1 = \tilde{x}_2 - \frac{l_{21}}{l_{22}}\tilde{x}_1 \tag{13.36}$$

[redacted]

$$y_3 = l_{31}x_1 + l_{32}x_2 + l_{33}x_3 + z_3. \tag{13.37}$$

[redacted]

$$x_3 = \tilde{x}_3 - \frac{l_{31}}{l_{33}}x_1 - \frac{l_{32}}{l_{33}}x_2 \tag{13.38}$$

[redacted]

$$\begin{bmatrix} x_1 \\ \tilde{x}_2 \\ \tilde{x}_3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \tilde{x}_1 \\ \tilde{x}_2 \\ \tilde{x}_3 \end{bmatrix}, \tag{13.39}$$

$$\begin{bmatrix} x_1 \\ x_2 \\ \tilde{x}_3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ -\frac{l_{21}}{l_{22}} & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ \tilde{x}_2 \\ \tilde{x}_3 \end{bmatrix}, \tag{13.40}$$

[redacted]

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -\frac{l_{31}}{l_{33}} & -\frac{l_{32}}{l_{33}} & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \tilde{x}_3 \end{bmatrix} \tag{13.41}$$

[redacted]

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -\frac{l_{31}}{l_{33}} & -\frac{l_{32}}{l_{33}} & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ -\frac{l_{21}}{l_{22}} & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \tilde{x}_1 \\ \tilde{x}_2 \\ \tilde{x}_3 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ -\frac{l_{21}}{l_{22}} & 1 & 0 \\ -\frac{l_{31}}{l_{33}} + \frac{l_{32}}{l_{33}}\frac{l_{21}}{l_{22}} & -\frac{l_{32}}{l_{33}} & 1 \end{bmatrix} \begin{bmatrix} \tilde{x}_1 \\ \tilde{x}_2 \\ \tilde{x}_3 \end{bmatrix}. \tag{13.42}$$

[redacted]

$$\begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} l_{11} & 0 & 0 \\ l_{21} & l_{22} & 0 \\ l_{31} & l_{32} & l_{33} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} z_1 \\ z_2 \\ z_3 \end{bmatrix}$$

$$= \begin{bmatrix} l_{11} & 0 & 0 \\ l_{21} & l_{22} & 0 \\ l_{31} & l_{32} & l_{33} \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ -\frac{l_{21}}{l_{22}} & 1 & 0 \\ -\frac{l_{31}}{l_{33}} + \frac{l_{32}}{l_{33}}\frac{l_{21}}{l_{22}} & -\frac{l_{32}}{l_{33}} & 1 \end{bmatrix} \begin{bmatrix} \tilde{x}_1 \\ \tilde{x}_2 \\ \tilde{x}_3 \end{bmatrix} + \begin{bmatrix} z_1 \\ z_2 \\ z_3 \end{bmatrix}$$

$$= \begin{bmatrix} l_{11} & 0 & 0 \\ 0 & l_{22} & 0 \\ 0 & 0 & l_{33} \end{bmatrix} \begin{bmatrix} \tilde{x}_1 \\ x_2 \\ \tilde{x}_3 \end{bmatrix} + \begin{bmatrix} z_1 \\ z_2 \\ z_3 \end{bmatrix} \tag{13.43}$$

[redacted]

$$\begin{bmatrix} 1 & 0 & 0 \\ -\frac{l_{21}}{l_{22}} & 1 & 0 \\ -\frac{l_{31}}{l_{33}} + \frac{l_{32}}{l_{33}}\frac{l_{21}}{l_{22}} & -\frac{l_{32}}{l_{33}} & 1 \end{bmatrix} = \begin{bmatrix} l_{11} & 0 & 0 \\ l_{21} & l_{22} & 0 \\ l_{31} & l_{32} & l_{33} \end{bmatrix}^{-1} \begin{bmatrix} l_{11} & 0 & 0 \\ 0 & l_{22} & 0 \\ 0 & 0 & l_{33} \end{bmatrix} \tag{13.44}$$

[redacted]

$$l_{u_1^* u_1^*} \geq l_{u_2^* u_2^*} \geq l_{u_3^* u_3^*} \geq l_{u_4^* u_4^*} \geq l_{uu} \tag{13.45}$$

[redacted]

[redacted]

$$c = x + 2A \cdot m \tag{13.46}$$

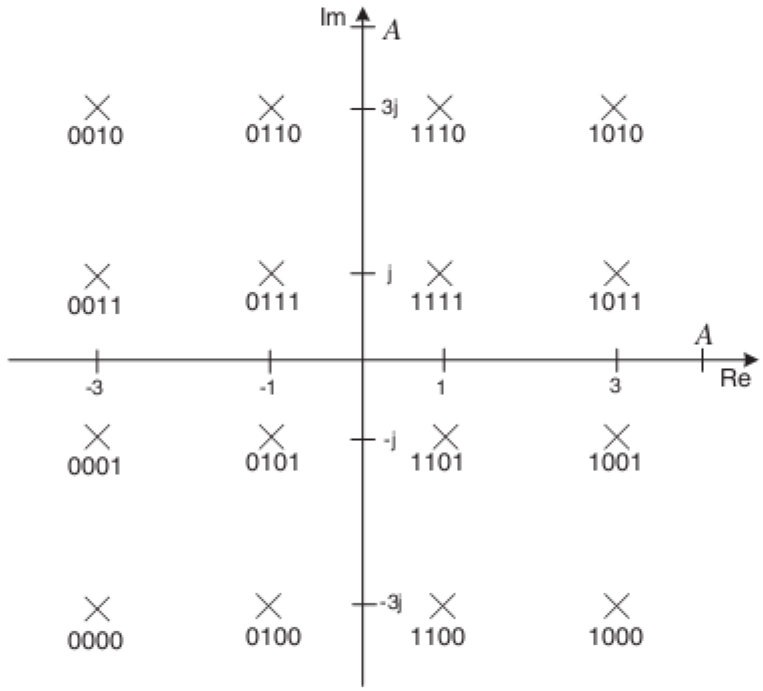
[redacted]

$$x = \text{mod}_A(c) \triangleq c - 2A\lfloor (c+A)/2A \rfloor \tag{13.47}$$

[redacted]

$$\text{mod}_A(x) = x - 2A\lfloor (x + A + jA)/2A \rfloor \tag{13.48}$$

[redacted]



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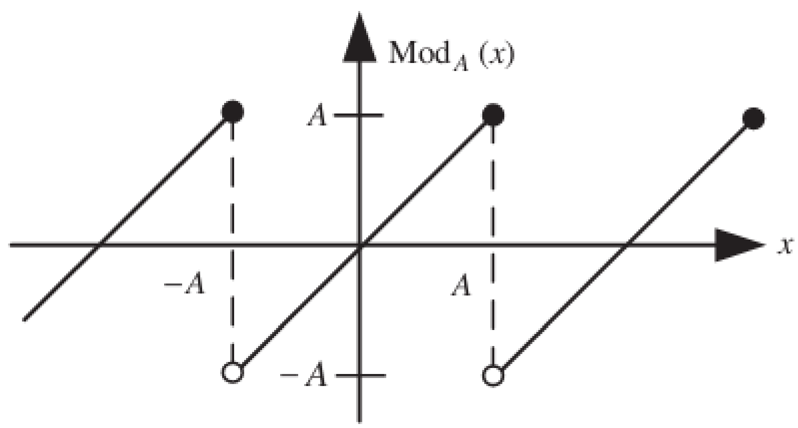


[redacted]

$$-A - jA \leq \mathrm{mod}_A(x) = x + 2A \cdot m + j\,2A \cdot m < A + jA \qquad (13.49)$$

[redacted]

$$x_1 < x_2 \Leftrightarrow \mathrm{Re}\{x_1\} < \mathrm{Re}\{x_2\} \text{ and } \mathrm{Im}\{x_1\} < \mathrm{Im}\{x_2\}. \qquad (13.50)$$

[redacted]

$$\mathrm{mod}_A(x) = x + 2A \cdot m + j\,2A \cdot n \qquad (13.51)$$

[redacted]

$$x_1^{\mathrm{TH}} = \mathrm{mod}_A(\tilde{x}_1) = \tilde{x}_1 \qquad (13.52)$$

$$x_2^{\mathrm{TH}} = \mathrm{mod}_A\left(\tilde{x}_2 - \frac{l_{21}}{l_{22}} x_1^{\mathrm{TH}}\right) \qquad (13.53)$$

$$x_3^{\mathrm{TH}} = \mathrm{mod}_A\left(\tilde{x}_3 - \frac{l_{31}}{l_{33}} x_1^{\mathrm{TH}} - \frac{l_{32}}{l_{33}} x_2^{\mathrm{TH}}\right) \qquad (13.54)$$

[redacted]

$$x_1^{\mathrm{TH}} = \tilde{x}_1 \qquad (13.55)$$

$$x_2^{\mathrm{TH}} = \tilde{x}_2 - \frac{l_{21}}{l_{22}} \tilde{x}_1 + 2A \cdot m_2 + j\,2A \cdot n_2 \qquad (13.56)$$

$$x_3^{\mathrm{TH}} = \tilde{x}_3 - \frac{l_{31}}{l_{33}} x_1^{\mathrm{TH}} - \frac{l_{32}}{l_{33}} x_2^{\mathrm{TH}} + 2A \cdot m_3 + j\,2A \cdot n_3 \qquad (13.57)$$

[redacted]

$$\begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \underbrace{\begin{bmatrix} \mathbf{H}_1^{\mathrm{DL}} \\ \mathbf{H}_2^{\mathrm{DL}} \\ \mathbf{H}_3^{\mathrm{DL}} \end{bmatrix}}_{\mathbf{H}^{\mathrm{DL}}} \mathbf{Q}^H \mathbf{x}^{\mathrm{TH}} + \begin{bmatrix} z_1 \\ z_2 \\ z_3 \end{bmatrix} = \begin{bmatrix} l_{11} & 0 & 0 \\ l_{21} & l_{22} & 0 \\ l_{31} & l_{32} & l_{33} \end{bmatrix} \begin{bmatrix} x_1^{\mathrm{TH}} \\ x_2^{\mathrm{TH}} \\ x_3^{\mathrm{TH}} \end{bmatrix} + \begin{bmatrix} z_1 \\ z_2 \\ z_3 \end{bmatrix} \tag{13.58}$$

[redacted]

$$y_2 = l_{21} x_1^{\mathrm{TH}} + l_{22} x_2^{\mathrm{TH}} + z_2 = l_{21} \tilde{x}_1 + l_{22} x_2^{\mathrm{TH}} + z_2. \tag{13.59}$$

[redacted]

$$\begin{aligned} y_2 &= l_{21}\tilde{x}_1 + l_{22}\left(\tilde{x}_2 - \frac{l_{21}}{l_{22}}\tilde{x}_1 + 2A \cdot m_2 + j\,2A \cdot n_2 \right) + z_2 \\ &= l_{22}(\tilde{x}_2 + 2A \cdot m_2 + j\,2A \cdot n_2) + z_2 \end{aligned} \tag{13.60}$$

[redacted]

$$\tilde{y}_2 = \frac{y_2}{l_{22}} = \tilde{x}_2 + 2A \cdot m_2 + j\,2A \cdot n_2 + \frac{z_2}{l_{22}} \tag{13.61}$$

[redacted]

$$\hat{\tilde{x}}_2 = \mathrm{mod}_A(\tilde{y}_2). \tag{13.62}$$

[redacted]

$$-A \leq \tilde{x}_2 + \frac{z_2}{l_{22}} < A, \tag{13.63}$$

[redacted]

$$\mathrm{mod}_A(\tilde{y}_2) = \tilde{y}_2 - 2A\left\lfloor \frac{(\tilde{y}_2 + A + jA)}{2A} \right\rfloor = \tilde{y}_2 - 2A(m_2 + jn_2) = \tilde{x}_2 + \frac{z_2}{l_{22}}. \tag{13.64}$$

[redacted]

$$y_3 = l_{31}s_1^{\mathrm{TH}} + l_{32}s_2^{\mathrm{TH}} + l_{33}s_3^{\mathrm{TH}} + z_3. \tag{13.65}$$

[redacted]

$$\begin{aligned} y_3 &= l_{31}x_1^{\mathrm{TH}} + l_{32}x_2^{\mathrm{TH}} + l_{33}x_3^{\mathrm{TH}} + z_3 \\ &= l_{31}x_1^{\mathrm{TH}} + l_{32}x_2^{\mathrm{TH}} + l_{33}\left(\tilde{x}_3 - \frac{l_{31}}{l_{33}}x_1^{\mathrm{TH}} - \frac{l_{32}}{l_{33}}x_2^{\mathrm{TH}} + 2A \cdot m_3 + j\,2A \cdot n_3\right) + z_3 \\ &= l_{33}(\tilde{x}_3 + 2A \cdot m_3 + j\,2A \cdot n_3) + z_3. \end{aligned} \tag{13.66}$$

[redacted]

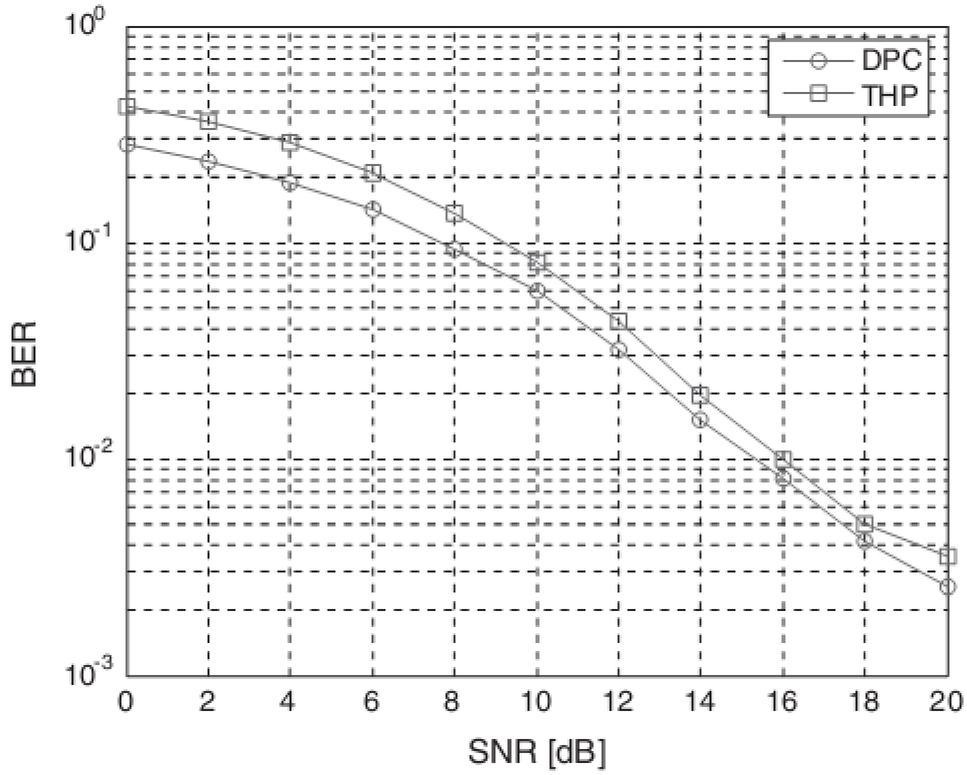
$$\hat{\tilde{x}}_3 = \mathrm{mod}_A(\tilde{y}_3) \tag{13.67}$$

[redacted]

$$\tilde{y}_3 = \frac{y_3}{l_{33}} = x_3 + 2A \cdot m_3 + j\,2A \cdot n_3 + \frac{z_3}{l_{33}}.$$

[redacted]

[redacted]



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